Time : 5 Hour | STD 11 Maths ch- 12 limits | Total Marks : 320

*** Choose the right answer from the given options. [1 Marks Each]** **[105]**

1. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots,$ then $\frac{dy}{dx} =$

(A) y + 1 (B) y - 1 (C) y (D) y^2

Ans. :

c. y

Solution:

$y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

Differentiate both the sides with respect to x, we get

$\frac{dy}{dx} = \frac{d}{dx}\left(y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\right)$

$= \frac{d}{dx}(1) + \frac{d}{dx}\left(\frac{x}{1!}\right) + \frac{d}{dx}\left(\frac{x^2}{2!}\right) + \frac{d}{dx}\left(\frac{x^3}{3!}\right) + \frac{d}{dx}\left(\frac{x^4}{4!}\right) + \ldots$

$= \frac{d}{dx}(1) + \frac{1}{1!}\frac{d}{dx}(x) + \frac{1}{2!}\frac{d}{dx}(x^2) + \frac{1}{3!}\frac{d}{dx}(x^3) + \frac{1}{4!}\frac{d}{dx}(x^4) + \ldots$

$= 0 + \frac{1}{1!} \times 1 + \frac{1}{2!} \times 2x + \frac{1}{3!} \times 3x^2 + \frac{1}{4!} \times 4x^3 + \ldots$

$= 1 + \frac{1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \left[\frac{n}{n!} = \frac{1}{(n-1)!}\right]$

$= y$

2. If $y = \frac{\sin x + \cos x}{\sin x - \cos x},$ then $\frac{dy}{dx}$ at x = 0 is:

(A) -2 (B) 0 (C) $\frac{1}{2}$ (D) does not exist

Ans. :

a. -2

Solution:

$y = \frac{\sin x + \cos x}{\sin x - \cos x}$

Differentiate both the sides with respect to x, we get

$= \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$

$= \frac{-(\cos^2 x + \sin^2 x - 2\cos x\sin x) - (\sin^2 x + \cos^2 x + 2\sin x\cos x)}{(\sin x - \cos x)^2}$

$= \frac{-1 + 2\cos x\sin x - 1 - 2\sin x\cos x}{(\sin x - \cos x)^2}$

$= \frac{-2}{(\sin x - \cos x)^2}$

Putting x = 0 is -2

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{-2}{(\sin 0 - \cos 0)} = \frac{-2}{(0-1)^2} = -2$$

Thus, $\frac{dy}{dx}$ at x = 0 is -2

3. Choose the correct answer.

$\lim\limits_{x \to \pi} \frac{x^2 \cos x}{1 - \cos x}$ is equal to:

(A) 2 (B) $\frac{3}{2}$ (C) $-\frac{3}{2}$ (D) 1

Ans. :

a. 2

Solution:

Given $\lim\limits_{x \to \pi} \frac{x^2 \cos x}{1 - \cos x} = \lim\limits_{x \to 0} \frac{x^2 \cos x}{2\sin^2 \frac{x}{2}}$

$$= \lim_{x \to 0} \frac{\frac{x^2}{4} \times 4\cos x}{2\sin^2 \frac{x}{2}} = \lim_{x \to 0} \frac{\left(\frac{x}{2}\right) \cdot 2\cos x}{\sin^2 \frac{x}{2}}$$

$$= \lim_{x \to 0} \left(\frac{\frac{x}{2}}{\sin \frac{x}{2}}\right) \cdot 2\cos x$$

$$= 2\cos x 0 = 2 \times 1 = 2$$

4. Choose the correct answer.

If $f(x) = \{\begin{matrix} x^2 - 1 & 0 < x < 2 \\ 2x + 3, & 2 \geq 3 < 3 \end{matrix}$ then the quadeatic equation whose roots are $\lim\limits_{x \to 2^-} f(x)$ and $\lim\limits_{x \to 2^+} f(x)$ is:

(A) $x^2 - 6x + 9 = 0$ (B) $x^2 - 7x + 8 = 0$

(C) $x^2 + 14x + 49 = 0$ (D) $x^2 - 10x + 21 = 0$

Ans. :

d. $x^2 - 10x + 21 = 0$

Solution:

$$\text{Given } f(x) = \begin{cases} x^2 - 1 & 0 < x < 2 \\ 2x + 3, & 2 \geq 3 < 3 \end{cases}$$

$$\therefore \lim_{x \to 2} f(x) = \lim_{x \to 2^-} (x^2 - 1)$$

$$\lim_{h \to 0} [(2 - x)^2 - 1] = \lim_{h \to 0} (4 + h^2 - 4h - 1)$$

$$= \lim_{h \to 0} (h^2 - 4h + 3) = 3$$

$$\therefore \lim_{x \to 2} f(x) = \lim_{x \to 2^+} (2x + 3)$$

$$= \lim_{h \to 0} [2(2 + h) + 3] = 7$$

Therefore, the quadratic equation whose roots are 3 and 7 is $x^2 - 10x + 21 = 0$

5. Find the derivative of e^{x2}:

(A) e^{x2} (B) 2^x (C) $2e^{x2}$ (D) $2xe^{x2}$

Ans. :

d. $2xe^{x2}$

Solution:

We apply chain rule.

First we differentiate x^2.

$\frac{d}{dx}(x2) = 2x$ Now, we know that $\frac{d}{dx}$(ex) = ex

We differentiate ex2 in the same manner and then,

multiply with the derivative of $\frac{x^2 d}{dx(e^{x2})} = 2xe^{x2}$.

6. $\lim_{x\to 0} x \sin \frac{1}{x}$ is equal to:

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) does not exist

Ans. :

a. 0

Solution:

We know that,

$= \lim_{x \to 0} x = 0$

And

$= -1 \leq \sin\frac{1}{x} \leq 1$

By Sandwich theorem,

$= \lim_{x \to 0} x\sin\frac{1}{x} = 0$

7. If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, then $\frac{dy}{dx}$ at x = 1 is

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 0

Ans. :

d. 0

Solution:

$y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$= x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

Differentiate both the sides with respect to x, we get

$\frac{1}{2}x^{\frac{1}{2}-1} + \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}$

$\frac{1}{2}x^{-\frac{1}{2}} - \left(\frac{1}{2}\right)x^{-\frac{3}{2}}$

Putting x = 1, we get

$\left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{2} \times 1 - \frac{1}{2} \times 1 = 0$

Thus, $\frac{dy}{dx}$ at x = 1 is 0.

8. Choose the correct answer.

$\lim\limits_{x \to 0} \frac{(\sqrt{x}-1)(2x-3)}{2x^2+x-3}$ is:

(A) $\frac{1}{10}$ (B) $-\frac{1}{10}$ (C) 1 (D) None of these.

Ans. :

b. $-\frac{1}{10}$

Solution:

Given $\lim\limits_{x \to 0} \frac{(\sqrt{x}-1)(2x-3)}{2x^2+x-3}$

$= \lim\limits_{x \to 1} \frac{(\sqrt{x}-1)(2x-3)}{x(2x+3)-1(2x+3)} = \lim\limits_{x \to 1} \frac{(\sqrt{x}-1)((2x-3)}{(x-1)(2x+3)}$

$= \lim\limits_{x \to 1} \frac{2x-3}{(\sqrt{x}+1)(2x+3)}$

Taking limit, we get

$= \frac{2(1)-3}{(\sqrt{x}+1)(2 \times 1+3)} = \frac{-1}{2 \times 5} = \frac{-1}{10}$

9. Evaluate the following limit $\lim_{x \to 0}\frac{1-\cos 2x}{x^2}$:

(A) 0 (B) 1 (C) 2 (D) None of these

Ans. :

c. 2

10. $\lim\limits_{x \to 0} \frac{\sin 5x}{\tan 3x}$

(A) $-\frac{5}{3}$ (B) $\frac{5}{3}$ (C) $-\frac{7}{3}$ (D) None of these

Ans. :

b. $\frac{5}{3}$

Solution:

$\lim\limits_{x \to 0} \frac{\sin 5x}{\tan 3x}$

$= \lim_{x\to 0} \frac{\sin 5x}{5x} \times \frac{3x}{\tan x} \times \frac{5}{3}$

we know that $= \lim_{x\to 0} \frac{\sin 5x}{5x} = 1$

$= \lim_{x\to 0} \times \frac{3x}{\tan x} = 1$

$= L = 1 \times 1 \times \frac{5}{3}$

$= \lim_{x\to 0} \frac{\sin 5x}{5x} = \frac{5}{3}$

11. Choose the correct answer.

$\lim_{x\to \pi} \frac{\sin x}{x-\pi}$ is:

(A) 1 (B) 2 (C) -1 (D) -2

Ans. :

c. -1

Solution:

Given, $\lim_{x\to \pi} \frac{\sin x}{x-\pi} = \lim_{x\to \pi} \frac{\sin(\pi)-x}{-(\pi-x)}$

$= -1$

12. Choose the correct answer.

$\lim_{x\to 0} \frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$ is:

(A) 2 (B) 0 (C) 1 (D) -1

Ans. :

c. 1

Solution:

Given $\lim_{x\to 0} \frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$

$= \lim_{x\to 0} \frac{\sin x[\sqrt{x+1}\sqrt{1-x}]}{(\sqrt{x+1}-\sqrt{1-x})(\sqrt{x+1}+\sqrt{1-x})}$

$= \lim_{x\to 0} \frac{\sin x[\sqrt{x+1}\sqrt{1-x}]}{x+1-1+x}$

$= \frac{1}{2} \cdot \lim_{x\to 0} \frac{\sin x}{x}\left[\sqrt{x+1} + \sqrt{1-x}\right]$

Taking limit, we get

$= \frac{1}{2} \times 1 \times \left[\sqrt{0+1} + \sqrt{0-1}\right] = \frac{1}{2} \times 1 \times 2$

$= 1$

13. $\lim_{x\to\infty} \sin x$ equals:

(A) 1 (B) 0 (C) ∞ (D) does not exist

Ans. :

d. does not exist

14. Find the derivative of e^{x^2}:

(A) e^{x^2} (B) 2x (C) $2e^{x^2}$ (D) $2xe^{x^2}$

Ans. :

d. $2xe^{x^2}$

Solution:

We apply chain rule. First we differentiate x^2.

$\frac{d}{dx}(x^2) = 2x$

Now, we know that $\frac{d}{dx}(e^x) = e^x$

We differentiate e^{x^2} in the same manner and then multiply with the derivative of x^2

$\frac{d}{dx}(e^x) = 2xe^x$

15. Given that f(x) is a differentiable function of x and that f(x) f(y) = f(x) + f(y) + f(xy) −2 and that f(2) = 5.Then f(3) is equal to?

(A) 6 (B) 24 (C) 15 (D) 19

Ans. :

a. 6

16. Choose the correct answer.

$\lim\limits_{x\to 0} \frac{\sec^2 x - 2}{\tan x - 1}$ is:

(A) 3 (B) 1 (C) 0 (D) 2

Ans. :

d. 2

Solution:

Given $\lim\limits_{x\to\frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1} = \lim\limits_{x\to\frac{\pi}{4}} \frac{1 + \tan^2 x - 2}{\tan x - 1}$

$= \lim\limits_{x\to\frac{\pi}{4}} \frac{\tan x - 1}{\tan x - 1} = \lim\limits_{x\to\frac{\pi}{4}} \frac{(\tan x + 1)(\tan x - 1)}{(\tan x - 1)}$

$= \lim\limits_{x\to\frac{\pi}{4}} (\tan x + 1) = \tan\frac{\pi}{4} + 1$

$= 1 + 1 = 2$

$= 2$

17. $\lim\limits_{x \to 1} (1 + \cos\pi)\cot^2 \pi x$:

(A) 1 (B) -1 (C) $\frac{1}{2}$ (D) 0

Ans. :

c. $\frac{1}{2}$

18. Choose the correct answer.

If $y = \frac{\sin x + \cos x}{\sin x - \cos x}$ then $\frac{dy}{dx}$ at x = 0 is equal to:

(A) -2 (B) 0 (C) $\frac{1}{2}$ (D) Does not exist.

Ans. :

a. -2

Solution:

Given $y = \frac{\sin x + \cos x}{\sin x - \cos x}$

$$\frac{dy}{dx} = \frac{-(\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$$

$$= \frac{-(\sin x + \cos x)^2 (\sin x + \cos x)}{(\sin x - \cos x)^2}$$

$$= \frac{\sin^2 x + \cos^2 x + 2\sin x\cos x}{(\sin x - \cos x)^2}$$

$$= \frac{-2}{(\sin x - \cos x)^2}$$

$$\therefore \left(\frac{dy}{dx}\right) = \frac{-2}{(\sin 0 - \cos 0)^2} = \frac{-2}{(-1)^2} = 2$$

19. f(x) = x − 1+ x − 3 then f (2) =:

(A) −2 (B) 2 (C) 0 (D) 1

Ans. :

c. 0

20. Choose the correct answer.

If $\begin{cases} \frac{\sin [x]}{[x]} & x \neq 0 \\ 0, & [x] = 0 \end{cases}$ where [.] denotes the greatest integer function. then $\lim\limits_{x \to 0} f(x)$ is equal to :

(A) 1 (B) 0 (C) -1 (D) None of these.

Ans. :

d. None of these.

Solution:

Given $\begin{cases} \frac{\sin[x]}{[x]} & x \neq 0 \\ 0, & [x] = 0 \end{cases}$

$\text{L. H. H} = \lim_{x \to 0} \frac{\sin[x]}{[x]} = \lim_{h \to 0} \frac{\sin[0-h]}{[0-h]}$

$= \lim_{h \to 0} \frac{-\sin[-h]}{[-h]} = -1$

$\text{R. H. L} = \lim_{x \to 0} \frac{\sin[x]}{[x]} = \lim_{h \to 0} \frac{\sin[0+h]}{[0+h]} = \lim_{h \to 0} \frac{\sin[h]}{[h]} = 1$

$\text{L. H. L} \neq \text{R. H. L}$

So, the limit does not exist.

21. Find the value of $\lim_{x \to 0} \frac{2x^2 + 3x + 4}{2}$

(A) 2 (B) 1 (C) $3\sqrt{5}$ (D) $2\sqrt{5}$

Ans. :

a. 2

Solution:

Let $\lim_{x \to 0} \frac{2x^2 + 3x + 4}{2}$ This is not an indeterminate form,

Therefore, $L = \lim_{x \to 0} \frac{2(0) + 3(0) + 4}{2} = \frac{4}{2} L = 2$

22. What is the value of ddx (sin x tan x)?

(A) sin x + tan x sec x

(B) cos x + tan x sec x

(C) sin x + tan x

(D) sin x + tan x $\sec^2$x

Ans. :

a. sin x + tan x sec x

Solution:

We follow product rule $\frac{d}{dx}(f. g) = g. \frac{d}{dx}(f) + (f)\frac{d}{dx}(f. g)$

Here, f = sin x and g = tan x

$\frac{d}{dx}$ (sin x tan x) = cos x tan x + $\sec^2$ x sinx

$\frac{d}{dx}$ (sin x tan x) = sin x + tan x sec x

23. $\lim_{x \to 0} \frac{\sin 7x}{\sin 3x}$ equals:

(A) $\frac{7}{3}$ (B) $\frac{10}{3}$ (C) $\frac{14}{3}$ (D) $\frac{1}{3}$

Ans. :

a. $\frac{7}{3}$

24. $\lim\limits_{n \to \infty} \frac{np\sin^2(n!)}{n+1}$, 0 < p < 1 is equal to:

(A) 0 (B) ∞ (C) 1 (D) None

Ans. :

a. 0

25. The value of $\lim\limits_{x \to 3^+} \frac{|x-3|}{x-3}$ equals:

(A) 1 (B) -1 (C) 0 (D) Does not exist

Ans. :

a. 1

Solution:

for $x = 30^+$,

$x-3 > 0$

Let $L = \lim\limits_{x-3^+} \frac{|x-3|}{x-3}$

$= \lim\limits_{x \to 3^+}$

$= \frac{(x-3)}{(x-3)}$

$= \lim\limits_{x \to 3^+} (1) = 1$

26. Choose the correct answer.

$\lim\limits_{x \to 0} \frac{1-\cos 4\theta}{1-\cos 6\theta}$ is equal to:

(A) $\frac{4}{9}$ (B) $\frac{1}{2}$ (C) $\frac{-1}{2}$ (D) -1

Ans. :

b. $\frac{4}{9}$

Solution:

Given $\lim\limits_{\theta \to 0} \frac{1-\cos 4\theta}{1-\cos 6\theta} = \lim\limits_{\theta \to 0} \frac{2\sin^2 2\theta}{2\sin^2 3\theta}$

$= \lim\limits_{\theta \to 0} \frac{\sin^2 2\theta}{\sin^2 3\theta} = \lim\limits_{\theta \to 0} \left[\frac{\sin 2\theta}{\sin 3\theta}\right]^2$

$= \lim\limits_{\theta \to 0} \left[\frac{\frac{\sin 2\theta}{2\theta} \times 2\theta}{\frac{\sin 3\theta}{2\theta} \times 3\theta}\right] = \left[\frac{2\theta}{2\theta}\right]^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$= \frac{4}{9}$

27. If $y = \dfrac{1+\dfrac{1}{x^2}}{1-\dfrac{1}{x^2}}$, then $\dfrac{dy}{dx} =$

(A) $-\frac{4x}{(x^2-1)^2}$ (B) $-\frac{4x}{x^2-1}$ (C) $\frac{1-x^2}{4x}$ (D) $\frac{4x}{x^2-1}$

Ans. :

a. $-\dfrac{4x}{(x^2-1)^2}$

Solution:

$$\mathbf{y = \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}}$$

$$= \frac{x^2+1}{x^2-1}$$

Differentiate both the sides with respect to x, we get

$$\frac{dy}{dx} = \frac{(x^2-1)\frac{d}{dx}(x^2+1)-(x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2} \text{ (Quotient rule)}$$

$$= \frac{(x^2-1)(2x+0)-(x^2+1)(2x-0)}{(x^2-1)^2}$$

$$= \frac{2x^3-2x-2x^3-2x}{(x^2-1)^2}$$

$$= \frac{-4x}{(x^2-1)^2}$$

28. Evaluate: $\lim x \to 2x^2 - 5x + 6$

(A) 1 (B) -5 (C) 0 (D) 4

Ans. :

c. 0

Solution:

$= \lim_{x\to 2} x^2 - 5x + 6$

$= 2^2 - 5 \times 2 + 6$

= 4 - 10 + 6

= 0

29. $\lim\limits_{x\to 0} \dfrac{t^x - e\sin x}{2(x-\sin x)} =$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$

Ans. :

a. $\dfrac{1}{2}$

30. Choose the correct answer.

$\lim\limits_{x \to 0} \frac{x^m - 1}{x^n - 1}$ is equal to:

(A) 1 (B) $\frac{m}{n}$ (C) $\frac{-m}{n}$ (D) m^2n^2

Ans. :

b. $\frac{m}{n}$

Solution:

Given $\lim\limits_{x \to 1} \frac{x^m - 1}{x^n - 1} = \lim\limits_{x \to 1} \frac{\frac{x^{m} - (1)^m}{x - 1}}{\frac{x^{n} - (1)^n}{x - 1}}$

$= \frac{m(1)^{m-1}}{n(1)^{n-1}} = \frac{m}{n}$

$= \frac{m}{n}$

31. Derivative of the function f(x) = (x - 1) (x - 2) is:

(A) 2x + 3 (B) 3x - 2 (C) 3x + 2 (D) 2x - 3

Ans. :

d. 2x - 3

32. If f(x) = 2x - 3, a = 2, l = 1f(x) = 2x - 3, a = 2, l = 1 and ϵ = 0.001 then δ > 0 satisfying 0<|x - a|< δ, | f(x) - l | < ϵ, is:

(A) 0.0050 (B) 0.0005 (C) 0.001 (D) 0.0001

Ans. :

b. 0.0005

Solution:

|f(x) - l|< 0.001 = ϵ ⇒ |2x - 3 - 1| (x) - l| < 0.001 ⇒ -0.001 < 2 x - 4 < 0.001

⇒ -0.0005 < x - 2 < 0.0005

⇒ |x - 2| < 0.0005

⇒ |x - a| < 0.0005 = δHence, δ = 0.0005 > 0

33. The limit of $\left[\frac{1}{x^2} + \frac{(2013)^x}{e^x - 1} - \frac{1}{e^x - 1}\right]$ as $x \to 0$:

(A) Approaches +∞ (B) Approaches −∞

(C) Is equal to $\log_e(2013)$ (D) Does not exist

Ans. :

a. Approaches +∞

34. Evaluate $\lim x \to 3\,(4x^2 + 3)$

(A) 36 (B) 39 (C) 40 (D) None of these

Ans. :

b. 39

Solution:

$$= \lim_{x \to 3} \left(4x^2 + 3\right) = 4(3)^2 + 3 = 36 + 3 = 39$$

35. $\lim \ \tan x = x \to \frac{\pi}{2}$

(A) 1 (B) 0 (C) $\frac{1}{\pi}$ (D) does not exisz

Ans. :

d. does not exisz

Solution:

$\text{L.H.L.} = \lim \ \tan x = +\infty \ x \to \left(\frac{\pi}{2}\right)^-$

$\text{R.H.L.} = \lim \ \tan x = -\infty \ x \to \left(\frac{\pi}{2}\right)^+$

Clearly left hand limit ≠ right hand limit.

Hence given limit does not exist.

36. If y = $5x^2$ + 8x find $\frac{dy}{dx}$

(A) 10x + 8 (B) 5x + 8 (C) $10x^2$ + 8x (D) None of these

Ans. :

a. 10x + 8

37. Let $3f(x) - 2f(\frac{1}{x}) = x$ then f(2) is equal to:

(A) $\frac{2}{7}$ (B) $\frac{1}{2}$ (C) 2 (D) 7

Ans. :

b. $\frac{1}{2}$

38. What is the value of the limit $f(x) = \frac{\sin^2 x + 2\sqrt{\sin x}}{x^2 - 4x}$ if x approaches 0?

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{-1}{\sqrt{2}}$ (C) $\frac{-1}{2\sqrt{2}}$ (D) $\frac{-1}{\sqrt{-2}}$

Ans. :

c. $\frac{-1}{2\sqrt{2}}$

Solution:

This is of the form $\frac{0}{0}$, therefore we use L 'Hospital' s rule and differentiate the numerator and denominator.

$$= \lim_{a \to b} \frac{2\text{sinx cos} + \cos x\sqrt{2}}{2x - 4x}$$

$$= \frac{0 + \sqrt{2}}{-4}$$

$$= \frac{-1}{2\sqrt{2}}$$

39. Choose the correct answer.

If f(x) = 1 - x + x^2 - x^3 + ...-x^{99} + x^{100}, then f'(1) is equal to:

(A) 150 (B) -50 (C) -150 (D) -50

Ans. :

d. 50

Solution:

Given that f(x) = 1 - x + x^2 - x^3 + ...-x^{99} + x^{100}

f'(x) = -1 + 2x - 3x^2 + ... -99.x^{98} + 100.x^{99}

f'(x) = -1 + 2 - 3 + ...-99 + 100

=(- 1 - 3 - 5 ....-99) + (2 + 4 + 6 + ...100)

$$= \frac{50}{2}\left[2 \times 1 + (50 - 1)(-2)\right] + \frac{50}{2}\left[2 \times 2(50 - 1)2\right]$$

$$= 25\left[-11 + 102\right] = 25 \times 2 = 50$$

40. Consider the differential equation $\frac{dy}{dx} = \cos x$ Then we observe that:

(A) $y = \sin x$ (B) $y = \sin x + 2$ (C) $y = \sin x - \frac{1}{2}$ (D) $y = \sin x + c$

Ans. :

d. $y = \sin x + c$

41. If,$y = (\sin^{-1}x)^2$, then what is the value of (1 - x^2)y - xy + 4?

(A) 2 (B) 4 (C) 6 (D) 8

Ans. :

c. 6

Solution:

We have, $y = (\sin^{-1}x)^2$ ....(1)

Differentiating with respect to x,

we get, $y = \frac{(\sin^{-1}x)^2}{1 - x^2}$ 1/2 or,

Squaring both sides,

(1 - x2)(y)2 = 4(sin - 1x)2 From (1),

(1 - x2)(y)2 = 4y Differentiating with respect to x,

4 = 2 + 4 = 6

42. What is the value of limy → 2y^2 - 4y - 2?

(A) 2 (B) 4 (C) 1 (D) 0

Ans. :

b. 4

Solution:

Explanation: $y^2 - 4 = (y - 2)(y + 2)$

herefore the fraction becomes, $(y + 2)$

As y tends to 2, the fraction becomes 4

43. The coefficient of y in the expansion of $(y^2 + \frac{c}{y})5$ is

(A) 10c (B) $10c^2$ (C) $10c^3$ (D) None of these

Ans. :

b. $10c^2$

Solution:

Given, binomial expression is $\left(y^2 + \frac{c}{y}\right)5$

Now, $T_{r+1} = {}^5C_r \times (y^2)^{5-r} \times \left(\frac{c}{y}\right)^r$

$= {}^5C_r \times y^{10-3r} \times C^r$

Now, $10 - 3r = 1$

$\Rightarrow 3r = 9$

$\Rightarrow r = 3$

So, the coefficient of y = ${}^5C_3 \times c^3 = 10c^3$

44. What is the value of $\text{limy}_{y \to \infty}\frac{2}{y}$?

(A) 0 (B) 1 (C) 2 (D) Infinity

Ans. :

a. 0

Solution:

Any number divided by infinity gives us 0.

Here, since the number 2 is divided by y,

as y approaches infinity, we get 0.

45. What is the value of the $\lim_{x \to 5}\frac{32x + 1}{x^2 - 5x}$?

(A) 6.2 (B) 6.4 (C) 6.3 (D) 6.1

Ans. :

b. 6.4

Solution:

Use L ’Hospital’ s Rule,

and differentiate the numerator and denominator.

$\lim_{x \to 5} \frac{32x+1}{x^2-5x}$?

$= \frac{32}{5}$

= 6.4

46. If $\lim\limits_{x \to 5} \frac{Xk\ \text{-}5K}{x-5} = 500$ then k is equal to:

(A) 3 (B) 4 (C) 5 (D) 6

Ans. :

b. 4

47. Evaluate: $\lim\limits_{x \to 1} \frac{2x^2 + 4x + 4}{2x - 1}$:

(A) 1 (B) 10 (C) 20 (D) 5

Ans. :

b. 10

Solution:

Given, $\lim\limits_{x \to 1} \frac{2x^2 + 4x + 4}{2x - 1}$:

Substituting x = 1 we get

$$\lim_{x \to 1} \frac{2x^2 + 4x + 4}{2x - 1} =$$

$$= \lim_{x \to 1} \frac{2(1)^2 + 4(1) + 4}{2(1) - 1}$$

$$= \lim_{x \to 1} \frac{2 + 4 + 4}{2 - 1} = 10$$

48. Choose the correct answer.

If $f(x) = \frac{x^n - a^n}{x - a}$ for some constant, a, then f'(a) is equal to:

(A) 1 (B) 0 (C) Does not exist (D) $\frac{1}{2}$

Ans. :

c. Does not exist

Solution:

Given $f(x) = \frac{x^n - a^n}{x - a}$

$f'(x) = \frac{(x-a)\,(n.x^{n\text{-}1} - (x^{n-a^n})\,1}{(x-a)^2}$

So, $f(a) = \frac{0}{0}$ = Does not exist.

49. The derivative of f(x) = $\sin^2 x$ is:

(A) cos 2x (B) tan 2x (C) sin 2x (D) cosec 2x

Ans. :

c. sin 2x

50. Choose the correct answer.

If $f(x) = \frac{x-4}{2\sqrt{x}}$ then f'(1) is equal to:

(A) $\frac{5}{4}$ (B) $\frac{4}{5}$ (C) 1 (D) 0

Ans. :

a. $\frac{5}{4}$

Solution:

Given that $f(x) = \frac{x-4}{2\sqrt{x}}$

$$\therefore f(x) = \frac{1}{2}\left[\frac{\sqrt{x}.1-(x-4).\frac{1}{2\sqrt{x}}}{x}\right]$$

$$= \frac{1}{2}\left[\frac{2x-x+4}{2\sqrt{x}.x}\right]$$

$$= \frac{1}{2}\left[\frac{x+4}{2(x)^{\frac{3}{2}}}\right]$$

$$\therefore f(x)\ x = 1 = \frac{1}{2}\left[\frac{1+4}{2\times 1}\right] = \frac{5}{4}$$

51. Identify the value of $\lim\limits_{x\to 2} x^2 - 5x + 6$

(A) 1 (B) -5 (C) 0 (D) 4

Ans. :

c. 0

Solution:

Let $\lim\limits_{x\to 2} x^2 - 5x + 6$ This is not an indeterminate form

Therefore, $L = (2)2 - 5(2) + 6 \Rightarrow L = 0.$

52. What is the derivative of $\lim\limits_{x\to\infty}(x\sin x(\frac{2}{x}))$?

(A) 2 (B) 1 (C) 3 (D) ∞

Ans. :

a. 2

53. f f(x) = 3cosx, then f ¢(x) at $x = \frac{\Pi}{2}$ = is:

(A) -3 (B) 3 (C) 0 (D) -1

Ans. :

a. -3

54. Choose the correct answer.

If $f(x) = 1 + x + \frac{x^2}{2} + \ldots\ldots + \frac{x^{100}}{100}$ then f'(1) is equal to:

(A) $\frac{1}{100}$ (B) 100 (C) does not exist (D) 0

Ans. :

b. 100

Solution:

Given $f(x) = 1 + x + \frac{x^2}{2} + \ldots\ldots + \frac{x^{100}}{100}$

$f(x) = 1 + \frac{2x}{2} + \ldots\ldots\ldots + \frac{100x}{100}$

$\therefore f'(1) = 1 + 11 + \ldots\ldots + 1 = 100$

55. If $\lim_{x \to 0} (\cos x + a\sin bx)^{\frac{1}{x}} = e^2$ then

the possible values of a & amp; bare:′a′&′b′are:

(A) a = 1, b = 2 (B) a = 2, b = 1 (C) a = 3, b = 2 (D) a = 2, b = 3

Ans. :

a. a = 1, b = 2

Solution:

$\lim_{x \to 0} (\cos x + a\sin bx)^{\frac{1}{x}}$ so its limit will be e^k, where

$k = \lim_{x \to 0} \frac{1}{x}(\cos x + a\sin bx - 1) = \lim_{x \to 0} \frac{-\sin x + ab\cos bx}{1} = ab = 2$

Hence all possible combination of aa and bb are possible whose product is 2

56. What is the value of ddx (sin x^3 cos x^2)?

(A) $3x^2$ cosx^2 cos x^3 + 2xsin x^3 sin x^2 (B) $3x^2$cos 2 cos x^3-2x sin x^3 sin x^2

(C) 2x cos x^2 cos x^3-2x sin x^3 sin x^2 (D) 2x cosx^2 cos x^3 + $3x^2$ sin x^3 sin x^2

Ans. :

b. 3x2 cos 2cosx3– 2xsin x3 sin x2

Solution:

We follow product rule $\frac{d}{dx}(f.g.) = g.\frac{d}{dx}(f) + f.\frac{dy}{dx}(g)$

Here $f = \sin x3$ and $g = \cos x2$

$\frac{d}{dx}(f) = 3x2 \cos x3$

$\frac{d}{dx}(g) = -2x \sin x^2$

We now substitute this in our main equation,

$= \cos x2 .3x2 \cos x3 + \sin x3.(-2x \sin x2)$

$= 3x2 \cos x2 \cos x3 – 2x \sin x3 \sin x2$

57. If f(x) = $x^{100} + x^{99}$ + ... + x + 1, then f(1) is equal to:

(A) 5050 (B) 5049 (C) 5051 (D) 50051

Ans. :

a. 5050

Solution:

f(x) = $x^{100} + x^{99}$ + ... + x + 1

f(x) = $100x^{99} + 99x^{98}$ + ... + 1 + 0

f(1) = $100(1)^{99} + 99(1)^{98}$ + ...+ 1

= 100 + 99 + ... + 1

This is an AP with common difference -1, a = 100, n = 100 and l = 1.

So, the sum of this AP = $\left(\frac{100}{2}\right)$[100 + 1]

= 50(101)

= 5050

Therefore, f(1) = 5050

58. $\lim_{x\to 3} 2x^2 - 3x - 5 =$

(A) 4 (B) 3 (C) -4 (D) -3

Ans. :

a. 4

Solution:

$\lim_{x\to 3} 2x^2 - 3x - 5$

$= 2(3)^2 - 3(3) - 5$

= 18 - 9 - 5

= 4

59. if $f(x) = x^{100} + x^{99} + \cdots + x + 1$, then f'(1) is equal to

(A) 5050 (B) 5049 (C) 5051 (D) 50051

Ans. :

a. 5050

Solution:

$f(x) = x^{100} + x^{99} + \cdots + x + 1$

Differentiate both the sides with respect to x, we get

$f^{'}(x) = \frac{d}{dx}(x^{100} + x^{99} + \cdots + x + 1)$

$= \frac{d}{dx}(x^{100}) + \frac{d}{dx}(x^{99}) + \cdots + \frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(1)$

$= 100x^{99} + 99x^{98} + \cdots + 2x + 1 + 0$

$= 100x^{99} + 99x^{98} + \cdots + 2x + 1$

Putting x = 1, we get

$f^{'}(x) = 100 + 99 + 98 + \cdots + 2 + 1$

$= \frac{100(100+1)}{2} \left(S_n = \frac{n(n+1)}{2}\right)$

$= 50 \times 101$

$= 5050$

60. What is the value of the limit $f(x) = x2 + \sqrt{2x}\sqrt{x2} - 4x$ if x approaches infinity?

(A) 0 (B) 2 (C) 5 (D) 4

Ans. :

a. 0

Solution:

This is of the form ∞, therefore we use L'Hospital's,
rule and differentiate the numerator and denominator.

61. Choose the correct answer.

If $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$ then $\frac{dy}{dx}$ at x = 1 is equal to:

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 0

Ans. :

d. 0

Solution:

Given that $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}}$

$\left(\frac{dy}{dx}\right) = \frac{1}{2} - \frac{1}{2} = 0$

62. What is the value of $\lim_{y \to \frac{\pi}{2}} \frac{\sin}{x}$?

(A) $\frac{2}{\pi}$ (B) $\frac{\pi}{2}$ (C) 1 (D) 0

Ans. :

a. $\frac{2}{\pi}$

Solution:

$\sin\frac{\pi}{2} = 1$

$$\lim_{y \to \frac{\pi}{2}} \frac{\sin}{x} = \frac{\sin \frac{\pi}{2}}{\frac{pi}{2}}$$

$$= \frac{1}{\frac{\pi}{2}}$$

$$= \frac{2}{\pi}$$

63. If $L = \lim_{x \to 0} \frac{a\sin x - \sin^2 x}{\tan^3 x}$ is finite, then the value of L is:

(A) 1 (B) 2 (C) 3 (D) -1

Ans. :

a. 1

64. What is the number of critical points for f(x) = max(sinx, cosx) for x belonging to (0, 2π)?

(A) 2 (B) 5 (C) 3 (D) 4

Ans. :

c. 3

Solution:

We know that in the range of (0, 2π)

the graph of sinx and cosx intersects each other in three points.

And we know that these points of intersection,

are only the critical points Thus, there are 3 critical points.

65. What is the value of $\frac{d}{dx}(e^x \tan x)$at $x = 0$?

(A) 0 (B) 1 (C) -1 (D) 2

Ans. :

b. 1

Solution:

We need to use product rule in both the terms to get the answer.

$\frac{d}{dx}(f.g) = g.\frac{d}{dx}(f) + (f).\frac{dy}{dx}(g)$

Here f = e^x and g = tan x

$\frac{d}{dx}(e^x \tan x) = \tan x.\frac{d}{dx}(e^x) + e^x.\frac{d}{dx}(\tan x)$

$\frac{d}{dx}(e^x \tan x) = \tan e^x + e^x.\sec 2x$

At x = 0 we get,

$= \tan 0.e0 + e0.\sec 20$

= 0.(1) + 1.(1)

= 1

66. What is the value of What is the value of $\lim_{x \to \infty} \frac{x^2 - 9}{x^2 - 3x + 2}$

(A) 1

(B) 2

(C) 0

(D) Limit does not exist

Ans. :

a. 1

Solution:

Since it is of the form $\frac{\infty}{\infty}$

we use L 'Hospital' s rule and differentiate the numerator and denominator

$L = \lim_{x \to \infty} \frac{x^2 - 9}{x^2 - 3x + 2}$

On differentiating once, we get $L = \lim_{x \to \infty} \frac{2x}{2x}$

Which is equal to, $\lim_{x \to \infty} 1 = 1.$

67. if $f(x) = 1 + x + \frac{x^2}{2} + \cdots + \frac{x^{100}}{100}$, then f'(1) is equal to:

(A) $\frac{1}{100}$

(B) 100

(C) 50

(D) 0

Ans. :

b. 100

Solution:

$f(x) = 1 + x + \frac{x^2}{2} + \cdots + \frac{x^{100}}{100}$

Differentiate both the sides with respect to x, we get

$f'(x) = \frac{d}{dx}\left(1 + x + \frac{x^2}{2} + \cdots + \frac{x^{100}}{100}\right)$

$= \frac{d}{dx}(1) + \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{x^2}{2}\right) + \cdots + \frac{d}{dx}\left(\frac{x^{100}}{100}\right)$

$= \frac{d}{dx}(1) + \frac{d}{dx}(x) + \frac{1}{2}\frac{d}{dx}(x^2) + \cdots + \frac{1}{100}\frac{d}{dx}(x^{100})$

$= 0 + 1 + \frac{1}{2} \times 2x + \cdots + \frac{1}{100} \times 100x^{99}$

$= 1 + x + x^2 + \cdots + x^{99}$

Putting x = 1, we get

$f'(x) = 1 + 1 + 1 + \cdots + 1$ (100 terms)

$= 100$

68. What is the value of $\lim_{y \to 4} f(y)$? It is given that f(y) = y^2 + 6y (y ≥ 2) and f(y) = 0(y < 2).

(A) 40

(B) 16

(C) 0

(D) 30

Ans. :

a. 40

Solution:

$\lim_{y\to 4} f(y) = y^2 + 6y$

$f(4) = 4^2 + 6(4)$

$f(4) = 16 + 24$

$f(4) = 40$

69. $\lim\limits_{x\to 1} (1 + \cos\pi)\cot^2 \pi x:$

(A) 1 (B) -1 (C) $\frac{1}{2}$ (D) 0

Ans. :

c. $\frac{1}{2}$

70. If $z_r = \cos\frac{r\alpha}{n^2} + i\sin\frac{r\alpha}{n^2}$ where r = 1, 2, 3, ...n then $\lim\limits_{n\to\infty} (z_1 . z_2 . . . . . z_n)$ is equal to:

(A) $\cos\frac{\alpha}{2}$ (B) $\sin\frac{\alpha}{2}$ (C) $e^{i\alpha}$ (D) $\sqrt{e^{i\alpha}}$

Ans. :

d. $\sqrt{e^{i\alpha}}$

71. What is the value of $(x + y)^2$ y if x = et sint and y = et cost?

(A) 12(y + y) (B) 2(y - y) (C) 2(xy + y) (D) 2(xy - y)

Ans. :

d. 2(xy - y)

Solution:

Since, x = et sint and y = et cost Therefore,

$\frac{dx}{dt}$ = e^t sint + e^t cost = y + x And,

$\frac{dx}{dt}$ = et cost - et sint = y - x So,

$= y = \frac{dx}{dt}$

$= \frac{(\frac{dy}{dt})}{(\frac{dx}{dt})}$

$= \frac{(y-x)}{(y+x)}$

Thus, $y = \frac{[(x+y)(y-1)-(y-x)(y+1)]}{(x+y)^2}$

Or, $(x + y)^2$ y = (x + y - y + x)y - x - y + x = 2xy - 2y = 2(xy - y) 10.

If, y = $(\sin\text{-}1x)^2$, then what is the value o.

72. Choose the correct answer.

If $y = \frac{\sin\ (x+9)}{\cos x}$ then $\frac{dy}{dx}$ at x = 0 is equal to:

(A) $\cos 9$ (B) $\sin 9$ (C) 0 (D) 1

Ans. :

a. $\cos 9$

Solution:

Given $y = \frac{\sin\ (x+9)}{\cos x}$

$$\frac{dy}{dx} = \frac{\cos x . \cos\ (x+9) - \sin\ (x+9)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos x \cos\ (x+9) + \sin x \sin\ (x+9)}{\cos^2 x}$$

$$= \frac{\cos\ (x+9-x)}{\cos^2 x} = \frac{\cos 9}{\cos^2 x}$$

$$= \frac{\cos 9}{(1)^2} = \cos 9$$

73. If $y = \frac{\sin\ (x+9)}{\cos x}$, then $\frac{dy}{dx}$ at x = 0 is:

(A) $\cos 9$ (B) $\sin 9$ (C) 0 (D) 1

Ans. :

a. $\cos 9$

Solution:

$y = \frac{\sin\ (x+9)}{\cos x}$

Differentiate both the sides with respect to x, we get

$$\frac{dy}{dx} = \frac{(\cos x)\frac{d}{dx}\sin\ (x+9) - \sin\ (x+9)\frac{d}{dx}(\cos x)}{\cos^2 x} \text{ (Quotient rule)}$$

$$= \frac{(\cos x)(\cos\ (x+9)) - (\sin\ (x+9))(-\sin x)}{\cos^2 x}$$

$$= \frac{(\cos x)(\cos\ (x+9)) + (\sin\ (x+9))(\sin x)}{\cos^2 x}$$

$$= \frac{\cos\ (x+9-x)}{\cos^2 x}$$

$$= \frac{\cos 9}{\cos^2 x}$$

Thus, $\frac{dy}{dx}$ at x = 0 is $\cos 9$

74. $\lim\limits_{x \to 0} \frac{ae^x + b\cos x + c.e^x}{\sin^2 x} = 4$ then b:

(A) 2 (B) 4 (C) 2 (D) -4

Ans. :

a. 2

75. What is the value $\lim_{x \to 4} \frac{x^2 - 2x - 8}{x - 4}$:

(A) 0 (B) 2 (C) 8 (D) 6

Ans. :

d. 6

Solution:

The denominator becomes 0, as x approaches 4.

$\lim_{x\to 4}\frac{x^2-2x-8}{x-4}$ Here, if we factorize the numerator we get

$\lim_{x\to 4}\frac{(x-4)(x+2)}{x-4}$

We can now cancel out (x - 4) from both the numerator and denominator.

We get, $\lim_{x\to 4}(x+2)=6$

76. Evaluate: $\lim\limits_{x\to 0}\frac{\sin x+\cos x}{\sin x-\cos x}$

(A) 0 (B) 1 (C) -1 (D) ∞

Ans. :

c. -1

Solution:

$\lim\limits_{x\to 0}\frac{\sin x+\cos x}{\sin x-\cos x}$

Substituting x = 0, we get

$=\lim\limits_{x\to 0}\frac{\sin x+\cos x}{\sin x-\cos x}$

$=\lim\limits_{x\to 0}\frac{\sin 0+\cos 0}{\sin 0-\cos 0}$

$=\lim\limits_{x\to 0}\frac{0+1}{0-1}$

77. What is the value of $\lim_{x\to 3}\frac{x^2-9}{x-3}$

(A) 0 (B) 3 (C) Infinity (D) 6

Ans. :

d. 6

Solution:

When x tends to 3, both the numerator and,

the denominator become 0 and it becomes of the form, 0.

Therefore, we use L'Hospital's rule,

which states the we differentiate the numerator and the denominator,

until a definite answer is reached.

On differentiating once we get,

$\lim_{x\to 3}\frac{2x}{1}$

Since, this not an indeterminate form now, we can substitute the value of x.

= 2 x 3

= 6

78. Choose the correct answer.

If $y = \dfrac{1 + \dfrac{1}{x^2}}{1 - \dfrac{1}{x^2}}$ then $\dfrac{dy}{dx}$ is equal to:

(A) $\dfrac{-4x}{(x^2-1)^2}$ (B) $\dfrac{-4x}{(x^2-1)^2}$ (C) $\dfrac{1-x^2}{4x}$ (D) $\dfrac{4x}{x^2-1}$

Ans. :

a. $\dfrac{-4x}{(x^2-1)^2}$

Solution:

Given, $y = \dfrac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}$

$\Rightarrow y = \dfrac{x^2+1}{x^2-1}$

$\therefore \dfrac{dy}{dx} = \dfrac{(x^2-1).2x-(x^2+1).2x}{(x^2-1)^2}$

$= \dfrac{2x(x^2-1-x^2-1)}{(x^2-1)^2} = \dfrac{2x(-2)}{(x^2-1)^2}$

$= \dfrac{-4x}{(x^2-1)^2}$

79. Derivative of the function f(x) = $7x^{-3}$ is:

(A) $21x^{-4}$ (B) $-21x^{-4}$ (C) $21x^4$ (D) $-21x^4$

Ans. :

b. $-21x^{-4}$

80. Evaluate: $\lim_{n\to\infty}\dfrac{n!}{(n+1)!-n!}$

(A) 0 (B) 1 (C) 2 (D) 3

Ans. :

a. 0

Solution:

We have, $\lim\limits_{n\to\infty} \dfrac{n!}{(n+1)!-n!}$

$= \lim\limits_{n\to\infty} \dfrac{n!}{(n+1)n!-n!}$

$= \lim\limits_{n\to\infty} \dfrac{1}{n+1-1}$

$$= \lim_{n \to \infty} \frac{1}{n} = 0$$

81. Choose the correct answer.

$\lim_{x \to 0} \frac{\operatorname{cosec} - \cot x}{x}$ is equal to:

(A) $-\frac{1}{2}$ (B) 1 (C) $\frac{1}{2}$ (D) -1

Ans. :

c. $\frac{1}{2}$

Solution:

Given $\lim_{x \to 0} \frac{\operatorname{cosec} x - \cot x}{x} = \lim_{x \to 0} \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x}$

$$= \lim_{x \to 0} \frac{1 - \cos x}{x \sin x} = \frac{2\sin^2 \frac{x}{2}}{x \cdot \sin \frac{x}{2} \cos \frac{x}{2}}$$

$$= \lim_{x \to 1} \frac{\sin \frac{x}{2}}{x \cos \frac{x}{2}} = \frac{\tan \frac{x}{2}}{x} = \lim_{x \to 0} \frac{\tan \frac{x}{2}}{2 \times \frac{x}{2}}$$

$$= \frac{1}{2} \times 1 = \frac{1}{2}$$

82. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = g(2)$, then $f^2(24)+g^2(24)$ is:

(A) 32 (B) 24 (C) 64 (D) 48

Ans. :

a. 32

83. The value of $\lim_{x \to a} \frac{\sqrt{x\text{-}b} - \sqrt{a\text{-}b}}{x^2 - a^2}$ $(a > b)$:

(A) $\frac{1}{4a}$ (B) $\frac{1}{a\sqrt{a-b}}$ (C) $\frac{2}{a\sqrt{a-b}}$ (D) $\frac{1}{4a\sqrt{a-b}}$

Ans. :

d. $\frac{1}{4a\sqrt{a-b}}$

Solution:

$$= \lim_{x \to a} \frac{\sqrt{x\text{-}b} - \sqrt{a\text{-}b}}{x^2 - a^2} \ (a > b):$$

This is the $\frac{0}{0}$ form.Apply L-hospital rule

$$= \lim_{x \to a} \frac{\frac{1}{2\sqrt{x-b}} - 0}{2x - 0}$$

$$\lim_{x \to a} \frac{1}{4x\sqrt{x-b}} = \frac{1}{4a\sqrt{a-b}}$$

Hence, this is the answer.

84. Choose the correct answer.

$\lim\limits_{x\to 0} \frac{\tan 2x - x}{3x - \sin x}$ is equal to:

(A) 2 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{4}$

Ans. :

b. $\frac{1}{2}$

Solution:

Given $\lim\limits_{x\to 0} \frac{\tan 2x - x}{3x - \sin x} = \lim\limits_{x\to 0} \frac{x\left[\frac{\tan 2x}{x} - 1\right]}{x\left[3 - \frac{\sin x}{x}\right]}$

$= \lim\limits_{x\to 0} \frac{\frac{\tan 2x}{2x} \times 2 - 1}{3 - \frac{\sin x}{x}} = \frac{1.2 - 1}{3 - 1}$

$= \frac{2-1}{2} = \frac{1}{2}$

85. If $f(x) = \frac{x^n - a^n}{x - a}$, then $f'(a)$ is:

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) dose not exist

Ans. :

d. dose not exist

Solution:

Given: $f(x) = \frac{x^n - a^n}{x - a}$

Now, f(x) is not difined at x = a. Therefore, f(x) is not differentiable at x = a.

So, f'(a) dose not exist.

Hence, the correct answer is option (d).

86. What is the value of $\lim_{y\to 0}(32x^2 \operatorname{cosec}^2 4x)$?

(A) 1 (B) 4 (C) 2 (D) 3

Ans. :

c. 2

Solution:

he limit can be written as,

$\lim_{x\to 0} \frac{32x^2}{\sin^2 4x}$

$2 \times \lim\limits_{x\to 0} \frac{4x}{\sin 4x} \times \lim\limits_{x\to 0} \frac{4x}{\sin 4x}$

= 2 x 1 x 1

= 2

87. If $f(x) = x\sin x$, then $f\left(\frac{\Pi}{2}\right)$ is equal to:

(A) 0 (B) 1 (C) 1 (D) $\frac{1}{2}$

Ans. :

b. 1

88. Choose the correct answer.

If f(x) = x^{100} + x^{99} .... + x + 1, then f'(1) is equal to:

(A) 5050 (B) 5049 (C) 5051 (D) 50051

Ans. :

a. 5050

Solution:

Given f(x) = x^{100} + x^{99} + ... + x + 1

f'(x) = 100.x^{100} + 99.x^{98} + ... + 1

S0, f'(1) = 100 + 99 + 98 + ... + 1

$= \frac{100}{2}[2 \times 100 + (100 - 1)(- 1)]$

$= 50[200 - 99] = 50 \times 101 = 5050$

$= 5050$

89. Let $f(x) = x - [x], x \in R$, then $f'\left(\frac{1}{2}\right)$ is:

(A) $\frac{3}{2}$ (B) 1 (C) 0 (D) -1

Ans. :

b. 1

Solution:

Given: $f(x) = x - [x], x \in R$

Now,

For $0 \le x < 1, [x] = 0$

$\therefore f(x) = x - 0 = x, \forall x \in [0, 1)$

Differentiate with respect to x, we get

$f'(x) = 1, \forall x \in [0, 1)$

$\therefore f'\left(\frac{1}{2}\right) = 1$

90. If $f(x) = 1 - x + x^2 - x^3 + \cdots - x^{99} + x^{100}$, then f'(1) equals

(A) 150 (B) -50 (C) -150 (D) 50

Ans. :

d. 50

Solution:

$f(x) = 1 - x + x^2 - x^3 + \cdots - x^{99} + x^{100}$

Differentiate both the sides with respect to x, we get

$f'(x) = \frac{d}{dx}(1 - x + x^2 - x^3 + \cdots - x^{99} + x^{100})$

$= \frac{d}{dx}(1) - \frac{d}{dx}(x) + \frac{d}{dx}(x^2) + \frac{d}{dx}(x^3) + \cdots - \frac{d}{dx}(x^{99}) + \frac{d}{dx}(x^{100})$

$= 0 - 1 + 2x - 3x^2 + \cdots - 99x^{98} + 100x^{99}$

Putting x = 1, we get

$f'(1) = -1 + 2 - 3 + \cdots - 99 + 100$

$= (-1 + 2) + (-3 + 4) + (-5 + 6) + \cdots + (-99 + 100)$

$= 1 + 1 + 1 + \cdots + 1$(50terms)

$= 50$

91. $\lim\limits_{x \to \pi} \frac{x^2 \cos x}{1 - \cos x}$ is equal to:

a. 2

b. $\frac{3}{2}$

c. $-\frac{3}{2}$

d. 1

Ans. :

a. 2

Solution:

Given $\lim\limits_{x \to \pi} \frac{x^2 \cos x}{1 - \cos x} = \lim\limits_{x \to 0} \frac{x^2 \cos x}{2\sin^2 \frac{x}{2}}$

$= \lim\limits_{x \to 0} \frac{\frac{x^2}{4} \times 4\cos x}{2\sin^2 \frac{x}{2}} = \lim\limits_{x \to 0} \frac{\left(\frac{x}{2}\right) \cdot 2\cos x}{\sin^2 \frac{x}{2}}$

$= \lim\limits_{x \to 0} \left(\frac{\frac{x}{2}}{\sin \frac{x}{2}}\right) \cdot 2\cos x$

$= 2\cos x 0 = 2 \times 1 = 2$

92. If $f(x) = \begin{cases} x^2 - 1 & 0 < x < 2 \\ 2x + 3, & 2 \geq 3 < 3 \end{cases}$ then the quadeatic equation whose roots are $\lim\limits_{x \to 2^-} f(x)$ and $\lim\limits_{x \to 2^+} f(x)$ is:

a. $x^2 - 6x + 9 = 0$

b. $x^2 - 7x + 8 = 0$

c. $x^2 + 14x + 49 = 0$

d. $x^2 - 10x + 21 = 0$

Ans. :

d. $x^2 - 10x + 21 = 0$

Solution:

Given $f(x) = \begin{cases} x^2 - 1 & 0 < x < 2 \\ 2x + 3, & 2 \geq 3 < 3 \end{cases}$

$\therefore \lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2^-} (x^2 - 1)$

$\lim\limits_{h \to 0} [(2 - x)^2 - 1] = \lim\limits_{h \to 0} (4 + h^2 - 4h - 1)$

$= \lim\limits_{h \to 0} (h^2 - 4h + 3) = 3$

$\therefore \lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2^+} (2x + 3)$

$= \lim\limits_{h \to 0} [2(2 + h) + 3] = 7$

Therefore, the quadratic equation whose roots are 3 and 7 is $x^2 - 10x + 21 = 0$

93. $\lim\limits_{x \to 0} \dfrac{(\sqrt{x} - 1)(2x - 3)}{2x^2 + x - 3}$ is:

a. $\dfrac{1}{10}$

b. $-\dfrac{1}{10}$

c. 1

d. None of these.

Ans. :

b. $-\dfrac{1}{10}$

Solution:

Given $\lim\limits_{x \to 0} \dfrac{(\sqrt{x} - 1)(2x - 3)}{2x^2 + x - 3}$

$= \lim\limits_{x \to 1} \dfrac{(\sqrt{x} - 1)(2x - 3)}{x(2x + 3) - 1(2x + 3)} = \lim\limits_{x \to 1} \dfrac{(\sqrt{x} - 1)((2x - 3)}{(x - 1)(2x + 3)}$

$= \lim\limits_{x \to 1} \dfrac{2x - 3}{(\sqrt{x} + 1)(2x + 3)}$

Taking limit, we get

$= \dfrac{2(1) - 3}{(\sqrt{x} + 1)(2 \times 1 + 3)} = \dfrac{-1}{2 \times 5} = \dfrac{-1}{10}$

94. If $y = \dfrac{\sin x + \cos x}{\sin x - \cos x}$ then $\dfrac{dy}{dx}$ at x = 0 is equal to:

a. -2

b. 0

c. $\dfrac{1}{2}$

d. Does not exist.

Ans. :

a. -2

Solution:

Given $y = \dfrac{\sin x + \cos x}{\sin x - \cos x}$

$$\frac{dy}{dx} = \frac{-(\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$$

$$= \frac{-(\sin x + \cos x)^2(\sin x + \cos x)}{(\sin x - \cos x)^2}$$

$$= \frac{\sin^2 x + \cos^2 x + 2\sin x\cos x}{(\sin x - \cos x)^2}$$

$$= \frac{-2}{(\sin x - \cos x)^2}$$

$$\therefore \left(\frac{dy}{dx}\right) = \frac{-2}{(\sin 0 - \cos 0)^2} = \frac{-2}{(-1)^2} = 2$$

95. $\lim\limits_{x \to 0} \dfrac{1 - \cos 4\theta}{1 - \cos 6\theta}$ is equal to:

a. $\dfrac{4}{9}$

b. $\dfrac{1}{2}$

c. $\dfrac{-1}{2}$

d. -1

Ans. :

b. $\dfrac{4}{9}$

Solution:

Given $\lim\limits_{\theta \to 0} \dfrac{1 - \cos 4\theta}{1 - \cos 6\theta} = \lim\limits_{\theta \to 0} \dfrac{2\sin^2 2\theta}{2\sin^2 3\theta}$

$$= \lim_{\theta \to 0} \frac{\sin^2 2\theta}{\sin^2 3\theta} = \lim_{\theta \to 0} \left[\frac{\sin 2\theta}{\sin 3\theta}\right]^2$$

$$= \lim_{\theta \to 0} \left[\frac{\frac{\sin 2\theta}{2\theta} \times 2\theta}{\frac{\sin 3\theta}{2\theta} \times 3\theta}\right] = \left[\frac{2\theta}{2\theta}\right]^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$= \frac{4}{9}$

96. $\lim\limits_{x \to 0} \frac{x^m - 1}{x^n - 1}$ is equal to:

a. 1

b. $\frac{m}{n}$

c. $\frac{-m}{n}$

d. $m^2 n^2$

Ans. :

b. $\frac{m}{n}$

Solution:

Given $\lim\limits_{x \to 1} \frac{x^m - 1}{x^n - 1} = \lim\limits_{x \to 1} \frac{\frac{x^{m - (1)^m}}{x - 1}}{\frac{x^{n - (1)^n}}{x - 1}}$

$= \frac{m(1)^{m-1}}{n(1)^{n-1}} = \frac{m}{n}$

$= \frac{m}{n}$

97. If f(x) = 1 - x + x^2 - x^3 + ...-x^{99} + x^{100}, then f'(1) is equal to:

a. 150

b. -50

c. -150

d. -50

Ans. :

d. 50

Solution:

Given that f(x) = 1 - x + x^2 - x^3 + ...-x^{99} + x^{100}

f'(x) = -1 + 2x - 3x^2 + ... -99.x^{98} + 100.x^{99}

f'(x) = -1 + 2 - 3 + ...-99 + 100

=(- 1 - 3 - 5 ....-99) + (2 + 4 + 6 + ...100)

$= \frac{50}{2}\left[2 \times 1 + (50 - 1)(-2)\right] + \frac{50}{2}\left[2 \times 2(50 - 1)2\right]$

$= 25\left[-11 + 102\right] = 25 \times 2 = 50$

98. If $f(x) = \frac{x^n - a^n}{x - a}$ for some constant, a, then f'(a) is equal to:

a. 1

b. 0

c. Does not exist

d. $\frac{1}{2}$

Ans. :

c. Does not exist

Solution:

Given $f(x) = \frac{x^n - a^n}{x - a}$

$f'(x) = \frac{(x-a)\,(n.x^{n\text{-}1} - (x^{n-a^n})\,1}{(x-a)^2}$

So, $f(a) = \frac{0}{0}$ = Does not exist.

99. If $f(x) = \frac{x-4}{2\sqrt{x}}$ then f'(1) is equal to:

a. $\frac{5}{4}$

b. $\frac{4}{5}$

c. 1

d. 0

Ans. :

a. $\frac{5}{4}$

Solution:

Given that $f(x) = \frac{x-4}{2\sqrt{x}}$

$\therefore f(x) = \frac{1}{2}\left[\frac{\sqrt{x}.1 - (x-4)\,.\,\frac{1}{2\sqrt{x}}}{x}\right]$

$= \frac{1}{2}\left[\frac{2x - x + 4}{2\sqrt{x.x}}\right]$

$= \frac{1}{2}\left[\frac{x+4}{2\,(x)^{\frac{3}{2}}}\right]$

$\therefore f(x)\ x = 1 = \frac{1}{2}\left[\frac{1+4}{2\times 1}\right] = \frac{5}{4}$

100. If $f(x) = 1 + x + \frac{x^2}{2} + \,.\,.\,.\,.\,.\, + \frac{x^{100}}{100}$ then f'(1) is equal to:

a. $\frac{1}{100}$

b. 100

c. does not exist

d. 0

Ans. :

b. 100

Solution:

Given $f(x) = 1 + x + \frac{x^2}{2} + \ldots\ldots + \frac{x^{100}}{100}$

$f(x) = 1 + \frac{2x}{2} + \ldots\ldots\ldots + \frac{100x}{100}$

$\therefore f'(1) = 1 + 11 + \ldots\ldots + 1 = 100$

101. If $y = \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}$ then $\frac{dy}{dx}$ is equal to:

a. $\frac{-4x}{(x^2-1)^2}$

b. $\frac{-4x}{(x^2-1)^2}$

c. $\frac{1-x^2}{4x}$

d. $\frac{4x}{x^2-1}$

Ans. :

a. $\frac{-4x}{(x^2-1)^2}$

Solution:

Given, $y = \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}$

$\Rightarrow y = \frac{x^2+1}{x^2-1}$

$\therefore \frac{dy}{dx} = \frac{(x^2-1).2x-(x^2+1).2x}{(x^2-1)^2}$

$= \frac{2x(x^2-1-x^2-1)}{(x^2-1)^2} = \frac{2x(-2)}{(x^2-1)^2}$

$= \frac{-4x}{(x^2-1)^2}$

102. $\lim\limits_{x\to 0} \frac{\text{cosec} - \cot x}{x}$ is equal to:

a. $-\frac{1}{2}$

b. 1

c. $\frac{1}{2}$

d. -1

Ans. :

c. $\frac{1}{2}$

Solution:

Given $\lim\limits_{x \to 0} \frac{\operatorname{cosec} x - \cot x}{x} = \lim\limits_{x \to 0} \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x}$

$= \lim\limits_{x \to 0} \frac{1 - \cos x}{x \sin x} = \frac{2\sin^2 \frac{x}{2}}{x \cdot \sin \frac{x}{2} \cos \frac{x}{2}}$

$= \lim\limits_{x \to 1} \frac{\sin \frac{x}{2}}{x \cos \frac{x}{2}} = \frac{\tan \frac{x}{2}}{x} = \lim\limits_{x \to 0} \frac{\tan \frac{x}{2}}{2 \times \frac{x}{2}}$

$= \frac{1}{2} \times 1 = \frac{1}{2}$

103. $\lim\limits_{x \to 0} \frac{(1+x)^n - 1}{x}$ is equal to:

a. n

b. 1

c. $-n$

d. 0

Ans. :

a. n

Solution:

Given $\lim\limits_{x \to 0} \frac{(1+x)^n - 1}{x} = \lim\limits_{x \to 0} \frac{(1+x)^n - (1)^n}{(1+x) - (1)}$

$= \lim\limits_{x \to 0} \frac{(1+x)^n - (1)^n}{1 + x - (2)} = n(1)^{n-1}$

$= n$

104. $\lim\limits_{x \to 0} \frac{\tan 2x - x}{3x - \sin x}$ is equal to:

a. 2

b. $\frac{1}{2}$

c. $-\frac{1}{2}$

d. $\frac{1}{4}$

Ans. :

b. $\frac{1}{2}$

Solution:

Given $\lim\limits_{x\to 0}\frac{\tan 2x - x}{3x - \sin x} = \lim\limits_{x\to 0}\frac{x\left[\frac{\tan 2x}{x} - 1\right]}{x\left[3 - \frac{\sin x}{x}\right]}$

$= \lim\limits_{x\to 0}\frac{\frac{\tan 2x}{2x}\times 2 - 1}{3 - \frac{\sin x}{x}} = \frac{1.2 - 1}{3 - 1}$

$= \frac{2-1}{2} = \frac{1}{2}$

105. If f(x) = $x^{100} + x^{99}$ .... + x + 1, then f'(1) is equal to:

a. 5050

b. 5049

c. 5051

d. 50051

Ans. :

a. 5050

Solution:

Given f(x) = $x^{100} + x^{99} + ... + x + 1$

f'(x) = $100.x^{100} + 99.x^{98} + ... + 1$

S0, f'(1) = 100 + 99 + 98 + ... + 1

$= \frac{100}{2}[2 \times 100 + (100 - 1)(- 1)]$

$= 50[200 - 99] = 50 \times 101 = 5050$

$= 5050$

*** Answer the following questions in one sentence. [1 Marks Each]** **[23]**

106. Find the derivative of

$(5x^3 + 3x - 1)(x - 1)$

Ans. : Let f(x) = $\left(5x^3 + 3x - 1\right)(x - 1)$

By product rule of differentiation,we have,

$f'(x) = \left(5x^3 + 3x - 1\right)\frac{d}{dx}(x - 1) + (x - 1)\frac{d}{dx}\left(5x^3 + 3x + 1\right)$

= $(5x^3 + 3x - 1) \times 1 + (x - 1) \times (15x^2 + 3)$

= $(5x^3 + 3x - 1) + (x - 1) \times (15x^2 + 3)$

$= 5x^3 + 3x - 1 + 15x^3 + 3x - 15x^2 - 3$

$= 20x^3 - 15x^2 + 6x - 4$

107. Find the derivative of x^{-3} (5 + 3x)

Ans. : Here $f(x) = x^{-3}(5 + 3x)$

$\therefore\ f'(x) = \frac{d}{dx}[x^{-3}(5 + 3x)]$

$= x^{-3}\frac{d}{dx}(5 + 3x) + (5 + 3x)\frac{d}{dx}(x^{-3})$

$= x^{-3} \times 3 + (5 + 3x) \times (-3x)^{-4}$

$= \frac{3}{x^3} - \frac{3}{x^4}(5 + 3x)$

$= \frac{3}{x^3}\left[1 - \frac{5+3x}{x}\right] = \frac{3}{x^3}\left[\frac{x-5-3x}{x}\right] = \frac{-3}{x^4}(5 + 2x)$

108. Find the derivative of x^5 $(3 - 6x^{-9})$

Ans. : Here $f(x) = x^5 (3 - 6x^{-9})$

$= x^5\frac{d}{dx}(3 - 6x^{-9}) + (3 - 6x^{-9})\frac{d}{dx}(x^5)$

$= x^5(54x^{-10}) + (3 - 6x^{-9}) \times 5x^4$

$= 54x^{-5} + 15x^4 - 30x^{-5}$

$= 24x^{-5} + 15x^4$

$= \frac{24}{x^5} + 15x^4$

109. Find the derivative of $x^{-4}(3 - 4x^{-5})$

Ans. : Here $f(x) = x^{-4} (3 - 4x^{-5})$

$f'(x) = \frac{d}{dx}[x^{-4} (3 - 4x^{-5})]$

$= x^{-4}\frac{d}{dx}(3 - 4x^{-5}) + (3 - 4x^{-5})\frac{d}{dx}(x^{-4})$

$= x^{-4} (20x^{-6}) + (3 - 4x^{-5}) (-4x^{-5})$

$= 20x^{-10} - 12x^{-5} + 16x^{-10}$

$= 36x^{-10} - 12x^{-5} = \frac{36}{x^{10}} - \frac{12}{x^5}$

110. Find the derivative of the function 5sec x + 4cos x

Ans. : Let f(x) = 5 sec x + 4 cos x

Therefore, we have

$\acute{f}(x) = \frac{d}{dx}(5\sec x + 4\cos x)$

$= 5\frac{d}{dx}(\sec x) + 4\frac{d}{dx}(\cos x)$

= 5 sec x tan x + 4 × (-sin x)

∴ f′(x) = 5 sec x tan x – 4 sin x

111. Find the derivative of function sin (x + 1) from first principle.

Ans. : Here f (x) = sin (x + 1)

Then f (x + h) = sin (x + h + 1)

We know that $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{\sin(x+h+1) - \sin(x+1)}{h}$

$= \lim\limits_{h \to 0} \frac{2\cos\left(\frac{2x+h+2}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$

$= \lim\limits_{h \to 0} \frac{\cos\left(x+1+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$ = cos (x + 1)

112. Find the derivative of function $f(x) = \cos\left(x - \frac{\pi}{8}\right)$ from first principle.

Ans. : Here $f(x) = \cos\left(x - \frac{\pi}{8}\right)$

$f(x) = \cos\left(x - \frac{\pi}{8}\right)$

Then f (x + h) = $\cos\left(x + h - \frac{\pi}{8}\right)$

We know that $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{\cos\left(x+h-\frac{\pi}{8}\right) - \cos\left(x-\frac{\pi}{8}\right)}{h}$

$= \lim\limits_{h \to 0} \frac{-2\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{h} = \lim\limits_{h \to 0} \frac{-\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)\cdot\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$

$= -\sin\left(x - \frac{\pi}{8}\right)$

113. Find the derivative of function $\frac{\sin(x+a)}{\cos x}$ (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers).

Ans. : Here $f(x) = \frac{\sin(x+a)}{\cos x}$

$\therefore\ f'(x) = \frac{d}{dx}\left[\frac{\sin(x+a)}{\cos x}\right]$

$= \frac{\cos x \frac{d}{dx}[\sin(x+a)] - \sin(x+a)\frac{d}{dx}(\cos x)}{\cos^2 x}$

$= \frac{\cos x . \cos(x+a) - \sin(x+a)(-\sin x)}{\cos^2 x}$

$= \frac{\cos x . \cos(x+a) + \sin x \sin(x+a)}{\cos^2 x}$

$= \frac{\cos(x+a-x)}{\cos^2 x}$ [$\because$ cos (A-B) = cos A cos B + sin A sin B]

$= \frac{\cos a}{\cos^2 x}$

114. Find the derivative of the function x^4 (5 sin x - 3 cos x).

Ans. : Let f(x) = $x^4(5\sin x - 3\cos x)$

By product rule of differentiation,we have,

$f'(x) = x^4\frac{d}{dx}(5\sin x - 3\cos x) + (5\sin x - 3\cos x)\frac{d}{dx}\left(x^4\right)$

$= x^4 \times \left[5\frac{d}{dx}(\sin x) - 3\frac{d}{dx}(\cos x)\right] + (5\sin x - 3\cos x)\frac{d}{dx}\left(x^4\right)$

$= x^4[5\cos x - 3(-\sin x)] + (5\sin x - 3\cos x)\left(4x^3\right)$

$\therefore f'(x) = x^3[5x\cos x + 3x\sin x + 20\sin x - 12\cos x]$

115. Find the derivative of function $(ax^2 + \sin x)(p + q\cos x)$ (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers).

Ans. : Here f(x) = (ax^2 + sinx) (p + q cosx)

$\therefore\ f'(x) = \frac{d}{dx}[(ax^2 + \sin x)(p + q\cos x)]$

$= (ax^2 + \sin x)\frac{d}{dx}(p + q\cos x) + (p + q\cos x)\frac{d}{dx}(ax^2 + \sin x)$

= (ax^2 + sin x) (-q sin x) + (p + q cos x) (2 ax + cos x)

= -q sin x (ax^2 + sin x) + (p + q cos x) (2ax + cos x).

116. Find the derivative of function $\frac{x}{1+\tan x}$ (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers).

Ans. : Here $f(x) = \frac{x}{1+\tan x}$

$\therefore\ f'(x) = \frac{d}{dx}\left[\frac{x}{1+\tan x}\right]$

$$= \frac{(1+\tan x)\frac{d}{dx}(x) - x\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$= \frac{(1+\tan x)(1) - x(\sec^2 x)}{(1+\tan x)^2} = \frac{1+\tan x - x\sec^2 x}{(1+\tan x)^2}$$

117. Find the limit: $\lim\limits_{x \to 1}[x^3 - x^2 + 1]$

Ans. : We have, $\lim\limits_{x \to 1}\left[x^3 - x^2 + 1\right]$ = $1^3 - 1^2 + 1 = 1$

118. Find the limit: $\lim\limits_{x \to -1}[1 + x + x^2 + \dots + x^{10}]$

Ans. : $\lim\limits_{x \to -1}\left[1 + x + x^2 + \dots + x^{10}\right] = 1 + (-1) + (-1)^2 + \dots + (-1)^{10}$

= 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 = 1

119. Find the limit: $\lim\limits_{x \to 2}[\frac{x^3 - 2x^2}{x^2 - 5x + 6}]$

Ans. : Evaluating the function at 2, we get it of the form $\frac{0}{0}$

Therefore, we have,

$$\lim_{x \to 2} \frac{x^3 - 2x^2}{x^2 - 5x + 6} = \lim_{x \to 2} \frac{x^2(x-2)}{(x-2)(x-3)}$$

$$= \lim_{x \to 2} \frac{x^2}{(x-3)} = \frac{(2)^2}{2-3} = \frac{4}{-1} = -4$$

120. Find the limit: $\lim\limits_{x \to 1}[\frac{x-2}{x^2 - x} - \frac{1}{x^3 - 3x^2 + 2x}]$

Ans. : First, we rewrite the function as a rational function.

$$\left[\frac{x-2}{x^2 - x} - \frac{1}{x^3 - 3x^2 + 2x}\right] = \left[\frac{x-2}{x(x-1)} - \frac{1}{x\left(x^2 - 3x + 2\right)}\right]$$

$$= \left[\frac{x-2}{x(x-1)} - \frac{1}{x(x-1)(x-2)}\right]$$

$$= \left[\frac{x^2 - 4x + 4 - 1}{x(x-1)(x-2)}\right]$$

$$= \frac{x^2 - 4x + 3}{x(x-1)(x-2)}$$

Evaluating the function at 1, we get it of the form $\frac{0}{0}$

Hence $\lim_{x \to 1}\left[\frac{x^2 - 2}{x^2 - x} - \frac{1}{x^3 - 3x^2 + 2x}\right] = \lim_{x \to 1} \frac{x^2 - 4x + 3}{x(x-1)(x-2)}$

$= \lim_{x\to 1}\frac{(x-3)(x-1)}{x(x-1)(x-2)}$

$= \lim_{x\to 1}\frac{x-3}{x(x-2)} = \frac{1-3}{1(1-2)} = 2$

121. Evaluate: $\lim\limits_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$

Ans. : We have,

$\lim\limits_{x\to 1}\frac{x^{15}-1}{x^{10}-1} = \lim\limits_{x\to 1}\frac{x^{15}-1}{x^{10}-1} \times \frac{(x-1)}{(x-1)}$

$= \lim\limits_{x\to 1}\frac{x^{15}-1}{x-1} \div \lim\limits_{x\to 1}\frac{x^{10}-1}{x-1}$

$\lim\limits_{x\to 1}\frac{x^{15}-(1)^{15}}{x-1} \div \lim\limits_{x\to 1}\frac{x^{10}-(1)^{10}}{x-1}$

$= 15(1)^{14} \div 10(1)^{9}$ [$\because \lim\limits_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$]

$= \frac{15}{10} = \frac{3}{2}$

122. Find the derivative at x = 2 of the function f(x) = 3x.

Ans. : We have $f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{3(2+h)-3(2)}{h}$

$= \lim_{h\to 0}\frac{6+3h-6}{h} = \lim_{h\to 0}\frac{3h}{h} = \lim_{h\to 0}3 = 3$

Therefore, derivative of the function 3x at x = 2 is 3.

123. Find the derivative of the function $f(x) = 2x^2 + 3x - 5$ at x = - 1. Also, prove that $f'(0) + 3f'(-1) = 0$.

Ans. : First, we find the derivatives of f(x) at x = -1 and x = 0. We have,

$f'(-1) = \lim\limits_{h\to 0}\frac{f(-1+h)-f(-1)}{h}$

$\left[\because f'(x) = \lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}\right]$

$= \lim\limits_{h\to 0}\frac{\left[2(-1+h)^2+3(-1+h)-5\right]-\left[2(-1)^2+3(-1)-5\right]}{h}$

$= \lim\limits_{h\to 0}\frac{\left[2\left(1+h^2-2h\right)-3+3h-5\right]-[2-3-5\}}{h}$

$= \lim\limits_{h\to 0}\frac{2h^2-h}{h} = \lim\limits_{h\to 0}(2h-1)$= 2(0) - 1 = -1

and $f'(0) = \lim\limits_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$\left[\because f'(x) = \lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}\right]$

$$= \lim_{h\to 0} \frac{\left[2(0+h)^2+3(0+h)-5\right]-\left[2(0)^2+3(0)-5\right]}{h}$$
$$= \lim_{h\to 0} \frac{2h^2+3h}{h}$$
$$= \lim_{h\to 0} (2h+3)$$
= 2(0) + 3 = 3

Now, f'(0) + 3f'(-1) = 3 - 3 = 0.

Hence proved.

124. Find the derivative of f(x) = x^2.

Ans. : $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h} = \lim_{h\to 0}(h+2x) = 2x$$

125. Find the derivative of f(x) = 1 + x + x^2 + x^3 + ... + x^{50} at x = 1.

Ans. : Given, f (a) = 1+ x + x^2 + x^3 +... + x^{50}

On differentiating both sides w.r.t. x, we get

f'(x) = 0 + 1 + 2x + $3x^2$ +... + $50x^{49}$

At x = 1,

f'(1) = 1 + 2(1)+ $3(1)^2$ + ... + $50(1)^{49}$

= 1 + 2 + 3 + ... + 50

$$= \frac{(50)(51)}{2} = 1275\left[\because \sum_n = \frac{n(n+1)}{2}\right]$$

126. Compute the derivative of f(x) = $\sin^2$ x.

Ans. : We have,

f(x) = $\sin^2$x

$$\therefore \quad \frac{df(x)}{dx} = \frac{d}{dx}(\sin x \sin x)$$

= (sin x)$\frac{d}{dx}$sinx + sin x$\frac{d}{dx}$(sinx) [using Leibnitz rule]

= (cos x)sinx + sinx(cos x)

= 2sin x cosx = sin 2x

127. Find the derivative of f from the first principle, where f is given by $f(x) = \frac{2x+3}{x-2}$

Ans. : Note that function is not defined at x = 2. But, we have

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{2(x+h)+3}{x+h-2}-\frac{2x+3}{x-2}}{h}$$
$$= \lim_{h\to 0}\frac{(2x+2h+3)(x-2)-(2x+3)(x+h-2)}{h(x-2)(x+h-2)}$$

$$= \lim_{h \to 0} \frac{(2x+3)(x-2) + 2h(x-2) - (2x+3)(x-2) - h(2x+3)}{h(x-2)(x+h-2)}$$

$$= \lim_{h \to 0} \frac{-7}{(x-2)(x+h-2)} = -\frac{7}{(x-2)^2}$$

128. Find the derivative of f from the first principle, where f is given by $f(x) = x + \frac{1}{x}$

Ans. : The function is not defined at x = 0. But, we have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\left(x + h + \frac{1}{x+h}\right) - \left(x + \frac{1}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[h + \frac{1}{x+h} - \frac{1}{x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[h + \frac{x-x-h}{x(x+h)}\right] = \lim_{h \to 0} \frac{1}{h}\left[h\left(1 - \frac{1}{x(x+h)}\right)\right]$$

$$= \lim_{h \to 0}\left[1 - \frac{1}{x(x+h)}\right] = 1 - \frac{1}{x^2}$$

*** Given section consists of questions of 2 marks each.** **[86]**

129. Evaluate $\lim\limits_{x \to 4} \frac{4x+3}{x-2}$

Ans. : Here $\lim\limits_{x \to 4} \frac{4x+3}{x-2} = \frac{4 \times 4 + 3}{4 - 2} = \frac{19}{2}$

130. Evaluate $\lim\limits_{x \to -1} \frac{x^{10} + x^5 + 1}{x - 1}$

Ans. : Here $\lim\limits_{x \to -1} \frac{x^{10} + x^5 + 1}{x - 1}$

$$= \frac{(-1)^{10} + (-1)^5 + 1}{-1-1} = \frac{1-1+1}{-2} = \frac{-1}{2}$$

131. Evaluate $\lim\limits_{x \to 0} \frac{(X+1)^5 - 1}{x}$

Ans. : Here $\lim\limits_{x \to 0} \frac{(X+1)^5 - 1}{x}$

$$= \lim_{x \to 0} \frac{(X+1)^5 - 1}{(x+1) - 1}$$

Putting x + 1 = y, as $x \to 0,\ y \to 1$

$$\therefore \lim_{y \to 0} \frac{y^5 - 1}{y - 1} = 5.(1)^{5-1}$$

$$= 5 \times 1 = 5\left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}\right]$$

132. Evaluate $\lim\limits_{x\to 2} \dfrac{3x^2 - x - 10}{x^2 - 4}$

Ans. : Here $\lim\limits_{x\to 2} \dfrac{3x^2 - x - 10}{x^2 - 4} \left[\dfrac{0}{0}\text{from}\right]$

$= \lim\limits_{x\to 2} \dfrac{(x-2)(3x+5)}{(x+2)(x-2)}$

$= \lim\limits_{x\to 2} \dfrac{3x-5}{x+2} = \dfrac{6+5}{2+2} = \dfrac{11}{4}$

133. Evaluate $\lim\limits_{x\to 3} \dfrac{x^4 - 81}{2x^2 - 5x - 3}$

Ans. : Here $\lim\limits_{x\to 3} \dfrac{x^4 - 81}{2x^2 - 5x - 3} \left[\dfrac{0}{0}\text{from}\right]$

$= \lim\limits_{x\to 3} \dfrac{(x^2+9)(x+3)(x-3)}{(x-3)(2x+1)}$

$= \lim\limits_{x\to 3} \dfrac{(x^2+9)(x+3)}{(2x+1)} = \dfrac{(3^2+9)(3+3)}{(2\times 3+1)} = \dfrac{108}{7}$

134. Evaluate $\lim\limits_{z\to 1} \dfrac{z^{1/3} - 1}{z^{1/6} - 1}$

Ans. : Here $\lim\limits_{z\to 1} \dfrac{z^{1/3} - 1}{z^{1/6} - 1} \left[\dfrac{0}{0}\text{form}\right]$

$= \lim\limits_{z\to 1} \dfrac{(z^{1/6}-1)^2}{z^{1/6}-1}$

$= \lim\limits_{z\to 1} \dfrac{(z^{1/6}+1)(z^{1/6}-1)}{(z^{1/6}-1)} = \lim\limits_{z\to 1}$ = $z^{1/6}$ + 1

= $(1)^{1/6}$ + 1 = 1 + 1 = 2

135. Evaluate $\lim\limits_{x\to 1} \dfrac{ax^2 + bx + c}{cx^2 + bx + a},\ a + b + c \neq 0$

Ans. : Here $\lim\limits_{x\to 1} \dfrac{ax^2 + bx + c}{cx^2 + bx + a}$

$= \dfrac{a\times(1)^2 + b\times 1 + c}{c\times(1)^2 + b\times 1 + a} = \dfrac{a+b+c}{c+b+a} = 1$

136. Evaluate $\lim\limits_{x\to -2} \dfrac{\dfrac{1}{x} + \dfrac{1}{2}}{x+2}$

Ans. : Here $\lim\limits_{x\to -2}\dfrac{\frac{1}{x}+\frac{1}{2}}{x+2}$

$=\lim\limits_{x\to -2}\dfrac{\frac{x+2}{2x}}{x+2}$

$=\lim\limits_{x\to -2}\dfrac{x+2}{2x}\times\dfrac{1}{x+2}$

$=\lim\limits_{x\to -2}\dfrac{1}{2x}=\dfrac{1}{2\times -2}=\dfrac{-1}{4}$

137. Evaluate $\lim\limits_{x\to 0}\dfrac{\sin ax}{bx}$

Ans. : Given, $\lim_{x\to 0}\frac{\sin ax}{bx}$

Applying the limits in the given expression we get, $\lim_{x\to 0}\frac{\sin ax}{bx}=\frac{0}{0}$

Multiplying and dividing the given expression by a we get,

$\Rightarrow \lim_{x\to 0}\frac{\sin ax}{bx}\times\frac{a}{a}$

$\Rightarrow \lim_{x\to 0}\frac{\sin ax}{ax}\times\frac{a}{b}$

We know that: $\lim_{x\to 0}\frac{\sin x}{x}=1$

$=\frac{a}{b}\lim_{ax\to 0}\frac{\sin ax}{ax}=\frac{a}{b}\times 1=\frac{a}{b}$

138. Evaluate $\lim\limits_{x\to 0}\dfrac{\sin ax}{\sin bx},\ a,\ b\neq 0$

Ans. : Here $\lim\limits_{x\to 0}\dfrac{\sin ax}{\sin bx}=\lim\limits_{x\to 0}\left[\dfrac{\sin ax}{ax}\times ax\times\dfrac{1}{\frac{\sin bx}{bx}\times bx}\right]$

$=\lim\limits_{x\to 0}\left[\dfrac{\sin ax}{ax}\times\dfrac{1}{\frac{\sin bx}{bx}}\times\dfrac{ax}{bx}\right]=\dfrac{a}{b}\lim\limits_{x\to 0}\left[\dfrac{\sin ax}{ax}\dfrac{1}{\frac{\sin bx}{bx}}\right]$

$=\frac{a}{b}\times 1\times 1=\frac{a}{b}$

139. Evaluate $\lim\limits_{x\to \pi}\dfrac{\sin(\pi-x)}{\pi(\pi-x)}$

Ans. : Let y= $\lim\limits_{x\to \pi}\dfrac{\sin(\pi-x)}{\pi(\pi-x)}\left[\dfrac{0}{0}\text{from}\right]$

Put $x=\pi+y$, as $x\to\pi,\ y\to 0$

$$\therefore y = \lim_{y \to 0} \frac{\sin\,[\pi - \pi - y]}{\pi[\pi - \pi - y]} = \lim_{y \to 0} \frac{\sin\,(-y)}{-\pi y}$$

$$= \lim_{y \to 0} \frac{-\sin y}{-\pi y} = \frac{1}{\pi} \lim_{y \to 0} \frac{\sin y}{y} = \frac{1}{\pi} \times 1 = \frac{1}{\pi}$$

140. Evaluate $\lim_{x \to 0} \frac{\cos x}{\pi - x}$

Ans. : Here $\lim_{x \to 0} \frac{\cos x}{\pi - x} = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$

141. Evaluate $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$

Ans. : Here $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1} = \lim_{x \to 0} \frac{1 - \cos 2x}{1 - \cos x}$

$$= \lim_{x \to 0} \frac{2\sin^2 x}{2\sin^2 x/2} = \lim_{x \to 0} \frac{(2\sin x/2\cos x/2)^2}{\sin^2 x/2}$$

$$= \lim_{x \to 0} \frac{4\sin^2 x/2\cos^2 x/2}{\sin^2 x/2} = \lim_{x \to 0} 4\cos^2 x/2 = 4/2 = 2$$

142. Evaluate $\lim_{x \to 0} \frac{ax + x\cos x}{b\sin x}$.

Ans. : We have, $\lim_{x \to 0} \frac{ax + x\cos x}{b\sin x} = \lim_{x \to 0} \left(\frac{ax}{b\sin x} + \frac{x\cos x}{b\sin x} \right)$

$$= \frac{a}{b} \lim_{x \to 0} \frac{x}{\sin x} + \frac{1}{b} \lim_{x \to 0} \frac{x\cos x}{\sin x}$$

$$= \frac{a}{b} \lim_{x \to 0} \frac{1}{\left(\frac{\sin x}{x} \right)} + \frac{1}{b} \lim_{bx \to 0} \frac{\cos x}{\left(\frac{\sin x}{x} \right)}$$

$$= \frac{a}{b} \frac{\lim_{x \to 0} 1}{\lim_{x \to 0} \left(\frac{\sin x}{x} \right)} + \frac{1}{b} \frac{\lim_{x \to 0} \cos x}{\lim_{x \to 0} \left(\frac{\sin x}{x} \right)} \left[\because \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \right]$$

$$= \frac{a}{b} \times \frac{1}{1} + \frac{1}{b} \times \frac{1}{1} \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \right]$$

$$= \frac{a + 1}{b}$$

143. Evaluate $\lim_{x \to 0} x\sec x$

Ans. : Here $\lim_{x \to 0} x\sec x$

$$= \lim_{x \to 0} x \times \frac{1}{\cos x} \to \lim_{x \to 0} \frac{x}{\cos x} = \frac{0}{1} = 0$$

144. Evaluate $\lim\limits_{x\to 0} \frac{\sin ax + bx}{ax + \sin bx}$; a, b, a + b $\neq$ 0.

Ans. : We have,

$$\lim_{x\to 0} \frac{\sin ax + bx}{ax + \sin bx} = \lim_{x\to 0} \frac{\frac{\sin ax}{x} + \frac{bx}{x}}{\frac{ax}{x} + \frac{\sin bx}{x}}$$

[dividing both numerator and denominator by x]

$$= \frac{\lim\limits_{x\to 0} \frac{\sin (ax)}{ax} \times a + \lim\limits_{x\to 0} b}{\lim\limits_{x\to 0} a + \lim\limits_{x\to 0} \frac{\sin bx}{bx} \times b}$$

$$\left[\because \lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} \text{ and } \lim_{x\to a} f(x) + g(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) \right]$$

$$= \frac{a \lim\limits_{x\to 0} \frac{\sin ax}{ax} + \lim\limits_{x\to 0} b}{\lim\limits_{x\to 0} a + b \lim\limits_{x\to 0} \frac{\sin bx}{bx}}$$

$$= \frac{a(1) + b}{a + b(1)} \left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1 \right]$$

$$= \frac{a+b}{a+b} = 1$$

145. Evaluate $\lim\limits_{x\to 0} (\operatorname{cosec} x - \cot x)$

Ans. : Here $\lim\limits_{x\to 0} (\operatorname{cosec} x - \cot x)$

$$= \lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$$

$$= \lim_{x\to 0} \frac{1 - \cos x}{\sin x}$$

$$= \lim_{x\to 0} \frac{2\sin^2 x/2}{2\sin x/2 \cos x/2} = \lim_{x\to 0} \tan x/2 = 0$$

146. Evaluate $\lim\limits_{x\to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

Ans. : Here $\lim\limits_{x\to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} \left[\frac{0}{0} \text{from} \right]$

Put $x = \frac{\pi}{2} + y$ as $x \to \frac{\pi}{2}$, $y \to 0$

$$\therefore \lim_{y\to 0} \frac{\tan 2\left(\frac{\pi}{2}+y\right)}{\frac{\pi}{2}+y-\frac{\pi}{2}} = \lim_{y\to 0} \frac{\tan\ (\pi+2y)}{y}$$

$$= \lim_{y\to 0} \frac{\tan 2y}{y} = \lim_{y\to 0} \frac{\tan 2y}{2y} \times 2 = 1 \times 2 = 2$$

147.

Evaluate $\lim_{x\to 0}$ f(x), where f(x) = $\begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Ans. : We have, f(x) = $\begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

LHL = $\lim_{x\to 0^-}$ f(x) = $\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{h\to 0} \frac{|0-h|}{(0-h)}$ [putting x = 0 - h as x $\to$ 0, then h $\to$ 0]

= $\lim_{h\to 0} \frac{|-h|}{-h}$

= $\lim_{h\to 0} \frac{h}{-h}$ [$\because$ |-x| = x]

= - 1

RHL = $\lim_{x\to 0^+}$ f(x) = $\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{h\to 0} \frac{|0+h|}{(0+h)} = \lim_{h\to 0} \frac{h}{h}$ = 1 [putting x = 0 + h as x $\to$ 0, then h $\to$ 0]

$\lim_{h\to 0^-}$ f(x) $\neq$ $\lim_{x\to 0^+}$ f(x)

Limit does not exist at x = 0

148.

Find $\lim_{x\to 0} f(x)$ where $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Ans. : Here $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

L.H.L. $= \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x}{|x|}$

Put x = 0 – h as x → 0, h → 0

$$\therefore \lim_{h\to 0} \frac{0-h}{|0-h|} = \lim_{h\to 0} \frac{-h}{|-h|} = \lim_{h\to 0} \frac{-h}{h} = -1$$

R.H.L. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} \frac{x}{|x|}$

Put x = 0 + h as x → 0, h → 0

$$\therefore \lim_{x\to 0} \frac{0+h}{|0+h|} = \lim_{n\to 0} \frac{h}{|h|} = \lim_{h\to 0} \frac{h}{h} = 1$$

Now L.H.L. $\neq$ R.H.L.

Thus limit does not exist at x = 0.

149. Find $\lim_{x \to 5} f(x)$, where f(x) = |x| - 5

Ans. : Here f(x) = |x| - 5

L.H.L. $\lim_{x \to 5} f(x) = \lim_{x \to 5^-} |x| - 5$

Put x = 5 – h as x → 5, h → 0

$\therefore \lim_{h \to 0} |5 - h| - 5 = \lim_{h \to 0} 5 - h - 5 = \lim_{h \to 0} (-h) = 0$

R.H.L. $= \lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} |x| - 5$

Put x = 5 + h as x → 5, h →0

$\therefore \lim_{h \to 0} |5 + h| - 5 = \lim_{h \to 0} 5 + h - 5 = \lim_{h \to 0} h = 0$

Now L.H.L. = R.H.L

Thus limit exists at x = 5 and $\lim_{x \to 0} f(x) = 0$

150. If the function f(x) satisfies $\lim_{x \to 1} \frac{f(x) - 2}{x^2 - 1} = \pi$, then evaluate $\lim_{x \to 1} f(x)$.

Ans. : Given, $\lim_{x \to 1} \frac{f(x) - 2}{x^2 - 1} = \pi \Rightarrow \frac{\lim_{x \to 1} [f(x) - 2]}{\lim_{x \to 1} \left(x^2 - 1\right)} = \pi$

$\Rightarrow \lim_{x \to 1} [f(x) - 2] = \pi \lim_{x \to 1} \left(x^2 - 1\right)$

$\Rightarrow \lim_{x \to 1} f(x) - 2 = \pi\left(1^2 - 1\right)$

$\Rightarrow \lim_{x \to 1} f(x) - 2 = \pi \times 0 \Rightarrow \lim_{x \to 1} f(x) - 2 = 0$

$\Rightarrow \lim_{x \to 1} f(x) = 2$

151. Find the derivative of 1/x^2 from the first principle.

Ans. : Here f$(x) = \frac{1}{x^2}$

Then f$(x + h) = \frac{1}{(x+h)^2}$

We know that f'$(x) = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow$ f'$(x) = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h \to 0} \frac{x^2 - (x+h)^2}{hx^2(x+h)^2}$

$= \lim_{h \to 0} \frac{x^2 - x^2 - h^2 - 2xh}{hx^2(x+h)^2} = \lim_{h \to 0} \frac{h(-h - 2x)}{hx^2(x+h)^2}$

$= \frac{-2x}{x^2 \times x^2} = \frac{-2}{x^3}$

152. Find the derivative of $\frac{x+1}{(x-1)}$ from the first principle.

Ans. : We have, $f(x) = \frac{x+1}{x-1}$

By first principle of derivative, we have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\left[\frac{(x+h)+1}{(x+h)-1} - \frac{x+1}{x-1}\right]}{h}$$

$$= \lim_{h \to 0} \frac{(x+h+1)(x-1) - (x+1)(x+h-1)}{h(x+h-1)(x-1)}$$

$$= \lim_{h \to 0} \frac{\left(x^2 + xh - h - 1\right) - \left(x^2 + xh + h - 1\right)}{h(x+h-1)(x-1)}$$

$$= \lim_{h \to 0} \frac{-2h}{h(x+h-1)(x-1)}$$

$$= \lim_{h \to 0} \frac{-2}{(x+h-1)(x-1)} = \frac{-2}{(x-1)^2}$$

153. For the function $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots + \frac{x^2}{2} + x + 1$ prove that f'(1) = 100f'(0)

Ans. : Here $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots + \frac{x^2}{2} + x + 1$

$$f'(x) = \frac{d}{dx}\left[\frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots + \frac{x^2}{2} + x + 1\right]$$

$$= \frac{1}{100}\frac{d}{dx}(x^{100}) + \frac{1}{99}\frac{d}{dx}(x^{99}) + \ldots + \frac{1}{2}\frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(1)$$

$$= \frac{1}{100} \times 100x^{99} + \frac{1}{99} \times 99x^{98} + \ldots + \frac{1}{2} \times 2x + 1 + 0$$

$= x^{99} + x^{98} + \ldots + x + 1$

Now $f'(1) = (1)^{99} + (1)^{98} + \ldots + (1) + 1 = 100$

$f'(0) = (0)^{99} + (0)^{98} + \ldots + 0 + 1 = 1$

Which shows that f'(1) = 100f'(0)

154. Find the derivative of $x^n + ax^{n-1} + a^2x^{n-2} + \ldots + a^{n-1}x + a^n$ for some fixed real number a.

Ans. : Let $f(x) = x^n + ax^{n-1} + a^2x^{n-2} + \ldots + a^{n-1}x + a^n$

On differentiating both sides, we get

$f'(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + \ldots + a^{n-1}.1 + 0$

On putting x = a both sides, we get

$f'(a) = na^{n-1} + a(n-1)a^{n-2} + a^2(n-2)a^{n-3} + \ldots + a^{n-1}$

$= n\,a^{n-1} + (n-1)\,a^{n-1} + (n-2)\,a^{n-1} + \ldots + a^{n-1}$

$= a^{n-1} [n + (n - 1) + (n - 2) + ... + 1]$

[$\because$ sum of n natural numbers $= \frac{n(n+1)}{2}$]

$f'(a) = \frac{n(n+1)}{2} a^{n-1}$

155. For some constants a and b, find the derivative of (x - a)(x - b)

Ans. : Here f (x) = (x - a)(x - b)

$\therefore \ f'(x) = \frac{d}{dx}(x-a)(x-b)$

$= (x-a)\frac{d}{dx}(x-b) + (x-b)\frac{d}{dx}(x-a)$

= (x - a) × 1 + (x - b) × 1

= x - a + x - b = 2x - a - b

156. Find the derivative of $\frac{x^n - a^n}{x-a}$ for some constant a.

Ans. : Here $f(x)\frac{x^n - a^n}{x-a}$

$\therefore \ f(x) = \frac{d}{dx}\left[\frac{x^n - a^n}{x-a}\right]$

$= \frac{(x-a)\frac{d}{dx}(x^n - a^n) - (x^n - a^n)\frac{d}{dx}(x-a)}{(x-a)^2}$

$= \frac{(x-a) \times nx^{n-1} - (x^n - a^n) \times 1}{(x-a)^2}$

$= \frac{nx^n - anx^{n-1} - x^n + a^n}{(x-a)^2}$

157. Find the derivative of the function $5\sin x - 6\cos x + 7$

Ans. : Let f(x) = 5 sin x – 6 cos x + 7

Therefore, we have,

$f'(x) = \frac{d}{dx}(5\sin x - 6\cos x + 7)$

$= 5\frac{d}{dx}(\sin x) - 6\frac{d}{dx}(\cos x) + \frac{d}{dx}(7)$

= 5 × cos x – 6 × (- sin x) + 0

$\therefore$ f′(x) = 5 cos x + 6 sin x

158. Find the derivative of function $\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$ (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers).

Ans. : Here $f(x) = \dfrac{1+\frac{1}{x}}{1-\frac{1}{x}} = \dfrac{x+1}{x-1}$

$f'(x) = \dfrac{d}{dx}\left[\dfrac{x+1}{x-1}\right]$

$= \dfrac{(x-1)\frac{d}{dx}(x+1)-(x+1)\frac{d}{dx}(x-1)}{(x-1)^2}$

$= \dfrac{(x-1)\times 1-(x+1)\times 1}{(x-1)^2}$

$= \dfrac{x-1-x-1}{(x-1)^2} = \dfrac{-2}{(x-1)^2},\ x \neq 0,\ 1.$

159. Find the derivative of function $\dfrac{1}{ax^2+bx+c}$(it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers).

Ans. : Here $f(x) = \dfrac{1}{ax^2+bx+c}$

$\therefore\ f'(x) = \dfrac{d}{dx}\left(\dfrac{1}{ax^2+bx+c}\right)$

$= \dfrac{(ax^2+bx+c)\frac{d}{dx}(1)-1.\frac{d}{dx}(ax^2+bx+c)}{(ax^2+bx+c)^2}$

$= \dfrac{(ax^2+bx+c)(0)-1(2ax+b)}{(ax^2+bx+c)^2} = \dfrac{-(2ax+b)}{(ax^2+bx+c)^2}$

160. Find the derivative of function $\dfrac{a}{x^4}-\dfrac{b}{x^2}+\cos x$ (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers).

Ans. : Here $f(x) = \dfrac{a}{x^4}-\dfrac{b}{x^2}+\cos x = ax^{-}4 - bx^{-2}+\cos x$

$\therefore\ f'(x) = \dfrac{d}{dx}[ax^{-4}-bx^2+\cos x] = a\dfrac{d}{dx}(x^{-4}) - b\dfrac{d}{dx}(x^{-2}) + \dfrac{d}{dx}(\cos x)$

$-ax^{-5}+2bx^{-3}-\sin x = \dfrac{-4a}{x^5}+\dfrac{2b}{x^3}-\sin x$

$-4ax^{-5}+2bx^{-3}-\sin x = \dfrac{-4a}{x^5}+\dfrac{2b}{x^3}-\sin x$

161. Find the derivative of function $4\sqrt{x}-2$ (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers).

Ans. : Here $f(x) = 4\sqrt{x}-2$

$\therefore\ f'(x) = \dfrac{d}{dx}\left[4\sqrt{x}-2\right]$

$= 4\dfrac{d}{dx}(\sqrt{x}) - \dfrac{d}{dx}(2)$

$= 4\times\dfrac{1}{2\sqrt{x}} - 0 = \dfrac{2}{\sqrt{x}}$

162. Find the derivative of function $\dfrac{\sin x+\cos x}{\sin x-\cos x}$ (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers).

Ans. : Here $f(x) = \dfrac{\sin x+\cos x}{\sin x-\cos x}$

$\therefore\ f'(x) = \dfrac{d}{dx}\left[\dfrac{\sin x+\cos x}{\sin x-\cos x}\right]$

$= \dfrac{(\sin x-\cos x)\frac{d}{dx}(\sin x+\cos x)-(\sin x+\cos x)\frac{d}{dx}(\sin x-\cos x)}{(\sin x-\cos x)^2}$

$= \dfrac{(\sin x-\cos x)(\cos x-\sin x)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}$

$= \dfrac{-(\sin x-\cos x)^2-(\sin x+\cos x)^2}{(\sin x-\cos x)^2}$

$= \dfrac{-(\sin^2 x-\cos^2 x+2\sin x\cos x-\sin^2 x-\cos^2 x-\cos^2 x-2\sin x\cos x}{(\sin x-\cos x)^2}$

$= \dfrac{-2(\sin^2 x+\cos^2 x)}{(\sin x-\cos x)^2} = \dfrac{-2}{(\sin x-\cos x)^2}$

163. Find the derivative of function $\dfrac{\sec x-1}{\sec x+1}$ (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers)

Ans. : $f(x) = \dfrac{\sec x-1}{\sec x+1}$

$\therefore\ f'(x) = \dfrac{d}{dx}\left[\dfrac{\sec x-1}{\sec x+1}\right]$

$= \dfrac{(\sec+1)\frac{d}{dx}(\sec x-1)-(\sec x-1)\frac{d}{dx}(\sec x+1)}{(\sec x+1)^2}$

$= \dfrac{(\sec x+1)(\sec x\tan x)-(\sec x-1)(\sec x\tan x)}{(\sec x+1)^2}$

$= \dfrac{\sec^2 x\tan x+\sec x\tan x-\sec^2 x\tan x+\sec x\tan x}{(\sec x+1)^2}$

$= \dfrac{2\sec x\tan x}{(\sec x+1)^2}$

164. Find the derivative of function $\dfrac{a+b\sin x}{c+d\cos x}$ (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers)

Ans. : Here $f(x) = \dfrac{a+b\sin x}{c+d\cos x}$

$\therefore\ f'(x) = \dfrac{d}{dx}\left[\dfrac{a+b\sin x}{c+d\cos x}\right]$

$= \dfrac{(c+d\cos x)\frac{d}{dx}(a+b\sin x)-(a+b\sin x)\frac{d}{dx}(c+d\cos x)}{(c+d\cos)^2}$

$= \dfrac{(c+d\cos x)(b\cos x)-(a+b\sin x)(-d\sin x)}{(c+d\cos x)^2}$

$$= \frac{bc\cos x + bd\cos^2 x + ad\sin x + bd\sin^2 x}{(c + d\cos x)^2}$$

$$= \frac{bc\cos x + ad\sin x + bd(\cos^2 x + \sin^2 x)}{(c + d\cos x)^2}$$

$$= \frac{bc\cos x + ad\sin x + bd}{(c + d\cos x)^2}$$

165. Find the derivative of the function $f(x) = \frac{4x + 5\sin x}{3x + 7\cos x}$

Ans. : Here $f(x) = \frac{4x + 5\sin x}{3x + 7\cos x}$

$$\therefore\ f'(x) = \frac{(3x + 7\cos x)\frac{d}{dx}(4x + 5\sin x) - (4x + 5\sin x)\frac{d}{dx}(3x + 7\cos x)}{(3x + 7\cos x)^2}$$

$$= \frac{(3x + 7\cos x)(4 + 5\cos x) - (4x + 5\sin x)(3 - 7\sin x)}{(3x + 7\cos x)^2}$$

$$= \frac{12x + 15x\cos x + 28\cos x + 35\cos^2 x - 12x + 28x\sin x - 15\sin x + 35\sin^2 x}{(3x + 7\cos x)^2}$$

$$= \frac{15x\cos x + 28\cos x + 28x\sin x - 15\sin x + 35(\cos^2 x + \sin^2 x)}{(3x + 7\cos x)^2}$$

$$= \frac{15x\cos x + 28\cos x + 28x\sin x - 15\sin x + 35}{(3x + 7\cos x)^2}$$

166. Find the derivative of the function $\frac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$ (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers)

Ans. : Let f(x) = $\frac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$

By quotient rule,we have,

$$f'(x) = \cos\frac{\pi}{4}\cdot\left[\frac{\sin x\frac{d}{dx}\left(x^2\right) - x^2\frac{d}{dx}(\sin x)}{\sin^2 x}\right]$$

$$= \cos\frac{\pi}{4}\cdot\left[\frac{\sin x \times 2x - x^2\cos x}{\sin^2 x}\right]$$

$$\therefore\ f'(x) = \frac{x\cos\frac{\pi}{4}[2\sin x - x\cos x]}{\sin^2 x}$$

167. Evaluate:

$$\lim_{x\to\frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$$

Ans. : Given that $\lim\limits_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$

$= \lim\limits_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right)}{x - \frac{\pi}{4}}$

$= \lim\limits_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\left(\cos \frac{\pi}{4}\sin x - \sin \frac{\pi}{4}\cos x\right)}{x - \frac{\pi}{4}}$

$= \lim\limits_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}}$

$= \sqrt{2}.1 = \sqrt{2}$

Hence, the required answer is $\sqrt{2}$.

168. Evaluate:

$$\lim_{x \to 0} \frac{1 - \cos 2x}{x^2}$$

Ans. : Given that $\lim\limits_{x \to 0} \frac{1 - \cos 2x}{x^2}$

$= \lim\limits_{x \to 0} \frac{2\sin^2 x}{x^2}$

$= \lim\limits_{x \to 0} 2\left(\frac{\sin x}{x}\right)^2$

$= 2 \times 1 = 2$

Hence, the required answer is 2.

169. Differentiate the following functions.

$$\frac{x^4 + x^3 + x^2 + 1}{x}$$

Ans. : $\frac{d}{dx}\left(\frac{x^4 + x^3 + x^2 + 1}{x}\right)$

$= \frac{d}{dx}\left(x^3 + x^2 + x + \frac{1}{x}\right)$

$= 3x^2 + 2x^2 + 1 - \frac{1}{x^2}$

Hence, the required answer is $3x^2 + 2x^2 + 1 - \frac{1}{x^2}$.

170. Evaluate:

$$\lim_{x \to 0} \frac{\sin x - 2\sin 3x + \sin 5x}{x}$$

Ans. : Given that $\lim_{x \to 0} \frac{\sin x - 2\sin 3x + \sin 5x}{x}$

$= \lim_{x \to 0} \frac{\sin x}{x} - \frac{2\sin 3x}{x} + \frac{\sin 5x}{x}$

$= \lim_{x \to 0} \frac{\sin x}{x} - \lim_{3x \to 0} 2(\frac{\sin 3x}{3x}) \times 3 + \lim_{5x \to 0} (\frac{\sin 5x}{5x}) \times 5$

$= 1 - 6 + 5 = 0$

Hence, the required answer is 0.

171. Find 'n' if $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80, x \in N$

Ans. : Given that $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$

$= n.(2)^{n-1} = 80$

$= n \times 2^{n-1} = 5 \times (2)^{5-1}$

$\therefore n = 5$

Hence, the required answer is n = 5.

*** Given section consists of questions of 3 marks each.** **[66]**

172. Find the derivative of $\frac{2}{x+1} - \frac{x^2}{3x-1}$

Ans. : Here $f(x) = \frac{2}{x+1} - \frac{x^2}{3x-1}$

$\therefore \ f'(x) = \frac{d}{dx}\left[\frac{2}{x+1} - \frac{x^2}{3x-1}\right] = \frac{d}{dx}\left(\frac{2}{x+1}\right) - \frac{d}{dx}\left(\frac{x^2}{3x-1}\right)$

$= \frac{(x+1)\frac{d}{dx}(2) - 2\frac{d}{dx}(x+1)}{(x+1)^2} - \frac{(3x-1)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(3x-1)}{(3x-1)^2}$

$= \frac{(x+1) \times 0 - 2 \times 1}{(x+1)^2} - \frac{(3x-1)(2x) - x^2 \times 3}{(3x-1)^2}$

$= \frac{-2}{(x+1)^2} - \frac{6x^2 - 2x - 3x^2}{(3x-1)^2} = \frac{-2}{(x+1)^2} - \frac{3x^2 - 2x}{(3x-1)^2}$

173. Find the derivative of function cosec x cot x.

Ans. : Here f (x) = cosec x cot x

$\therefore$ f'(x) = $\frac{d}{dx}$ [cosec x cot x]

= cosec x $\frac{d}{dx}$ (cot x) + cot x $\frac{d}{dx}$ (cosec x)

= cosec x . – cosec^2 x + cot x . – cosec x cot x

= - cosec^3 x – cosec x $\cot^2$ x.

174. Evaluate:

$$\lim_{x \to \frac{\pi}{3}} \frac{\sqrt{1-\cos 6}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)}$$

Ans. : Given that $\lim_{x \to \frac{\pi}{3}} \frac{\sqrt{1-\cos 6}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)}$

$$= \lim_{x \to \frac{\pi}{3}} \frac{\sqrt{2\sin^2 3x}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)}$$

$$= \lim_{x \to \frac{\pi}{3}} \frac{\sqrt{2}\sin 3x}{\sqrt{2}\left(\frac{\pi-3x}{3}\right)}$$

$$= \lim_{x \to \frac{\pi}{3}} \frac{3.\sin\ (\pi-3x)}{\pi-3x}$$

$$= 3\left[\therefore \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

Hence, the required answer is 3.

175. Evaluate:

$$\lim_{x \to 3} \frac{x^3+27}{x^5+243}$$

Ans. : Given that $\lim_{x \to 3} \frac{x^3+27}{x^5+243}$

$$= \lim_{x \to 3} \frac{\frac{x^3+(3)^3}{x-3}}{\frac{x^5+(3)^3}{x-3}}$$ [Dividing the Nr and Den. By x - 3]

$$= \lim_{x \to 3} \frac{\frac{x^3-(3)^3}{x+3}}{\frac{x^5-(3)^5}{x+3}}$$

$$= \frac{3(-3)^{3-1}}{5(-3)^{5-1}}$$

$$= \frac{3\times(-3)^2}{5\times(-3)^4}$$

$$= \frac{1}{5\times 3} = \frac{1}{15}$$

Hence, the required answer is $\frac{1}{15}$.

176. Evaluate:

$$\lim_{x \to 1} \frac{x^7-2x^5+1}{x^3-3x^2+2}$$

Ans. : Given that $\lim\limits_{x \to 1} \dfrac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

$= \lim\limits_{x \to 1} \dfrac{x^7 - x^5 - x^5 + 1}{x^3 - x^2 - 2x^2 + 2}$

$= \lim\limits_{x \to 1} \dfrac{x^5(x^2 - 1) - 1(x^5 - 1)}{x^2(x - 1) - 2(x^2 - 1)}$

Dividing the numerator and denominator by (x - 1) we get

$= \lim\limits_{x \to 1} \dfrac{x^5\left(\frac{x^2-1}{x-1}\right) - 1\left(\frac{x^5-1}{x-1}\right)}{x^2\left(\frac{x-1}{x-1}\right) - 2\left(\frac{x^2-(1)^5}{x-1}\right)}$

$= \dfrac{\lim\limits_{x \to 1} x^5(x+1) - \lim\limits_{x \to 1}\left(\frac{x^5-(1)^5}{x-1}\right)}{\lim\limits_{x \to 1} x^2 - 2\lim\limits_{x \to 1}(x+1)}$

$= \dfrac{1(2) - 5.(1)^{5-1}}{1 - 2(2)}$

$= \dfrac{2-5}{1-4} = \dfrac{-3}{-3} = 1$

Hence, the required answer is 1.

177. Evaluate:

$$\lim_{x \to 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2}$$

Ans. : Given that $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2}$

$= \lim\limits_{x \to 0} \dfrac{\left[\sqrt{1+x^3} - \sqrt{1-x^3}\right]\left[\sqrt{1+x^3} + \sqrt{1-3^3}\right]}{x^2\left[\sqrt{1+x^3} + \sqrt{1-x^3}\right]}$

$= \lim\limits_{x \to 0} \dfrac{(1+x^3) - (1-x^3)}{x^2\left[\sqrt{1+x^3} + \sqrt{1-x^3}\right]}$

$= \lim\limits_{x \to 0} \dfrac{1+x^3-1-x^3}{x^2\left[\sqrt{1+x^3} + \sqrt{1-x^3}\right]}$

$= \lim\limits_{x \to 1} f(x)$

Hence, the required answer is 0.

178. Evaluate:

$$\lim_{x \to \sqrt{2}} \frac{x^2 - 4}{x^2 + 3\sqrt{2x} - 8}$$

Ans. : Given that $\lim\limits_{x \to \sqrt{2}} \dfrac{x^2 - 4}{x^2 + 3\sqrt{2x} - 8}$

$= \lim\limits_{x \to \sqrt{2}} \dfrac{(x^2 - 2)(x^2 + 2)}{x^2 + 4\sqrt{2}x - \sqrt{2}x - 8}$

$$= \lim_{x \to \sqrt{2}} \frac{(x+\sqrt{2})(x-\sqrt{2})(x^2+2)}{x(x+4\sqrt{2})-\sqrt{2}(x+4\sqrt{2})}$$

$$= \lim_{x \to \sqrt{2}} \frac{(x+\sqrt{2})(x-\sqrt{2})(x^2+2)}{(x+4\sqrt{2})(x-\sqrt{2})}$$

$$= \lim_{x \to \sqrt{2}} \frac{(x+\sqrt{2})(x^2+2)}{x+4\sqrt{2}}$$

Taking limits we have

$$= \frac{(\sqrt{2}+\sqrt{2})(2+2)}{\sqrt{2}+4\sqrt{2}}$$

$$= \frac{2\sqrt{2} \times 4}{5\sqrt{2}} = \frac{8}{5}$$

Hence, required answre is $\frac{8}{5}$.

179. If $\lim\limits_{x \to 1} \frac{x^4-1}{x-1} = \lim\limits_{x \to k} \frac{x^3-k^3}{x^2-k^2}$ then find the value of K.

Ans. : Given that $\lim\limits_{x \to 1} \frac{x^4-1}{x-1} = \lim\limits_{x \to k} \frac{x^3-k^3}{x^2-k^2}$

$$\Rightarrow 4(1)^{4-1} = \lim_{x \to k} \frac{(x-k)(x^2+k^2+kx)}{(x-k)(x+k)}$$

$$\Rightarrow 4 = \lim_{x \to k} \frac{(x-k)(x^2+k^2+kx)}{(x-k)(x+k)}$$

$$\Rightarrow 4 = \frac{k^2+k^2+k^2}{2k}$$

$$\Rightarrow 4 = \frac{3k^2}{2k}$$

$$\Rightarrow 4 = \frac{3}{2}k$$

$$\Rightarrow k = \frac{8}{3}$$

Hence, the required value of k is $\frac{8}{3}$.

180. Evaluate:

$$\lim_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

Ans. : Given that $\lim\limits_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

$$= \lim_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h\left[\sqrt{x+h}+\sqrt{x}\right]} \times \sqrt{x+h}+\sqrt{x}$$

$$= \lim_{h \to 0} \frac{x+h-x}{h\left[\sqrt{x+h}+\sqrt{x}\right]}$$

$$= \lim_{h \to 0} \frac{h}{h\left[\sqrt{x+h}+\sqrt{x}\right]}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

Talking the limits, We have

$$\frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Hence, the answer is $\frac{1}{2\sqrt{x}}$.

181. Evaluate:

$$\lim_{x \to a} \frac{(2+x)\frac{5}{2} - (a+2)\frac{5}{2}}{x-a}$$

Ans. : Given that $\lim_{x \to a} \frac{(2+x)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x-a}$

$$= \lim_{x \to a} \frac{(2+x)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{(2+x)-(a+2)}$$

$$= \lim_{2+x \to a+2} \frac{(2+x)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{(2+x)-(a+2)}$$

$$= \frac{5}{2}(a+2)^{\frac{5}{2}-1}$$

$$= \frac{5}{2}(a+2)^{\frac{3}{2}}$$

Hence, the required answre is $\frac{5}{2}(a+2)^{\frac{3}{2}}$.

182. Differentiate the following functions.

$$\frac{x^5 - \cos x}{\sin x}$$

Ans. : $\frac{d}{dx}\left(\frac{x^5-\cos x}{\sin x}\right)$

$$= \frac{\sin x\frac{d}{dx} - (x^5 - \cos x)\,.\,\frac{d}{dx}(\sin x)}{\sin^2 x}$$

$$= \frac{\sin x(5x^4 + \sin x) - (x^5 - \cos x)(\cos x)}{\sin^2 x}$$

$$= \frac{5x^4 . \sin x + \sin^2 x - x^5 \cos x + \cos^2 x}{\sin^2 x}$$

$$= \frac{5x^4 . \sin x - x^5 \cos x + (\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{5x^4 \sin x - x^5 \cos x + 1}{\sin^2 x}$$

Hence, the required answer is $\frac{5x^4 \sin x - x^5 \cos x + 1}{\sin^2 x}$.

183. Evaluate:

$$\lim_{x \to 1} \frac{x^4 - \sqrt{x}}{\sqrt{x} - 1}$$

Ans. : Given that $\lim\limits_{x \to 1} \frac{x^4 - \sqrt{x}}{\sqrt{x} - 1}$

$$= \lim_{x \to 1} \frac{\sqrt{x}\left[(x)^{\frac{7}{2}} - 1 \right]}{\sqrt{x - 1}}$$

$$= \lim_{x \to 1} \frac{\sqrt{x}\frac{\left[x^{\frac{7}{2}} - (1)^{\frac{7}{2}} \right]}{x - 1}}{\frac{(x)^{\frac{1}{2}} - (1)^{\frac{1}{2}}}{x - 1}}$$

Dividing the numerator and denominator of x - 1

$$= \lim_{x \to 1} \frac{\sqrt{x}\frac{\left[x^{\frac{7}{2}} - (1)^{\frac{7}{2}} \right]}{x - 1}}{\frac{(x)^{\frac{1}{2}} - (1)^{\frac{1}{2}}}{x - 1}} \times \lim_{x \to 1} \sqrt{x}$$

$$= \frac{\frac{7}{2}(1)^{\frac{7}{2} - 1}}{\frac{1}{2}(1)^{\frac{1}{2} - 1}} \times \sqrt{1}$$

$$= \frac{\frac{7}{2}}{\frac{1}{2}} = 7$$

Hence, the required answer is 7.

184. Evaluate:

$$\lim_{x \to \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$$

Ans. : Given that $\lim\limits_{x \to \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$

$$= \lim_{x \to \frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 1 - 3}{\operatorname{cosec} x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(\operatorname{cosec} x + 2)(\operatorname{cosec} x - 2)}{(\operatorname{cosec} x - 2)}$$

$$= \lim_{x \to \frac{\pi}{6}} (\operatorname{cosec} x + 2)$$

Taking limit we have

$$= \operatorname{cosec}\frac{\pi}{6} + 2 = 2 + 2 = 4$$

Hence, the required answer is 4.

185. Evaluate:

$$\lim_{x \to 0} \frac{1-\cos mx}{1-\cos nx}$$

Ans. : Given that $\lim_{x \to 0} \frac{1-\cos mx}{1-\cos nx}$

$$= \lim_{x \to 0} \left(\frac{2\sin^2 \frac{m}{2}x}{2\sin^2 \frac{n}{2}x}\right)$$

$$= \lim_{x \to 0} \left(\frac{\sin \frac{m}{2}x}{\sin \frac{n}{2}x}\right)^2$$

$$= \frac{\lim_{x \to 0} \left(\frac{\sin \frac{m}{2}x}{\frac{m}{2}x} \times \frac{m}{2}x\right)^2}{\lim_{x \to 0} \left(\frac{\sin \frac{n}{2}x}{\frac{n}{2}x} \times \frac{n}{2}x\right)^2}$$

$$= \frac{1.\frac{m^2}{4}x^2}{1.\frac{n^2}{4}x^2}$$

$$= \frac{m^2}{n^2}$$

Hence, the required answer is $\frac{m^2}{n^2}$.

186. Evaluate:

$$\lim_{x \to 0} \frac{(x+2)^{\frac{1}{3}} - 2^{\frac{1}{3}}}{x}$$

Ans. : Given that $\lim_{x \to 0} \frac{(x+2)^{\frac{1}{3}} - 2^{\frac{1}{3}}}{x}$

Put x + 2 = y

⇒ x= y - 2

$$= \lim_{y-2 \to 0} \frac{y^{\frac{1}{3}} - 2^{\frac{1}{3}}}{y-2}$$

$$= \lim_{y \to 2} \frac{y^{\frac{1}{3}} - 2^{\frac{1}{3}}}{y-2}$$

$$= \frac{1}{3}.(2)^{\frac{1}{3}-1} = \frac{1}{3}.2^{\frac{-2}{3}}$$

Hence, the answer is $\frac{1}{3}.(2)^{\frac{-2}{3}}$.

187. Evaluate:

$$\lim_{x \to 0} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$$

Ans. : Given that $\lim\limits_{x \to 0} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$

Dividing the numerator and denominator by x, we get

$$= \lim\limits_{h \to 0} \frac{\frac{(1+x)^6 - 1}{x}}{\frac{(1+x)^2 - 1}{x}}$$

Putting 1 + x = y

⇒ x = y - 1

$$= \lim\limits_{h \to 0} \frac{\frac{y^6 - (1)^6}{y - 1}}{\frac{y^2 - (1)^2}{y - 1}}$$

$$= \lim\limits_{h \to 0} \frac{\frac{y^6 - (1)^6}{y - 1}}{\frac{y^2 - (1)^2}{y - 1}}$$

$$\frac{6.(1)^{6-1}}{2.(1)^{2-1}} = \frac{6}{2}$$

$$= 3$$

Hence, the required qnswer is 3.

188. Evaluate:

$$\lim\limits_{x \to 0} \frac{\sin 3x}{\sin 7x}$$

Ans. : Given that $\lim\limits_{x \to 0} \frac{\sin 3x}{\sin 7x}$

$$= \lim\limits_{x \to 0} \frac{\frac{\sin 3x}{3x} \times 3x}{\frac{\sin 7x}{7x} \times 7x}$$

$$= \frac{\lim\limits_{3x \to 0} \left(\frac{\sin 3x}{3x} \right)}{\lim\limits_{7x \to 0} \left(\frac{\sin 7x}{7x} \right)} \times \frac{3}{7}$$

$$= \frac{1}{1} \times \frac{3}{7} = \frac{3}{7}$$

Hence, the required answer is $\frac{3}{7}$.

189. Differentiate the following functions.

$$\frac{a + b\sin x}{c + d\cos x}$$

Ans. : $\frac{d}{dx} \left(\frac{a + b\sin x}{c + d\cos x} \right)$

$$= \frac{(c + d\cos x) \cdot \frac{d}{dx}(a + b\sin x) - (a + b\sin x) \frac{d}{dx}(c + d\cos x)}{(c + d\cos x)^2}$$

$$= \frac{cb\cos x + bd\cos^2 x + ad\sin x + bd\sin^2 x}{(c + d\cos x)^2}$$

$$= \frac{cb\cos x + ad\sin x + bd(\cos^2 x + \sin^2 x)}{(c + d\cos x^2)}$$
$$= \frac{cb\cos x + ad\sin x + bd}{(c + d\cos x)^2}$$

190. Differentiate the following functions.

$$\frac{x^5 - \cos x}{\sin x}$$

Ans. : $\frac{d}{dx}\left(\frac{x^5 - \cos x}{\sin x}\right)$

$$= \frac{\sin x\frac{d}{dx} - (x^5 - \cos x)\,.\,\frac{d}{dx}(\sin x)}{\sin^2 x}$$
$$= \frac{\sin x(5x^4 + \sin x - (x^5 - \cos x)(\cos x)}{\sin^2 x}$$
$$= \frac{5x^4\,.\sin x + \sin^2 x - x^5\cos x + \cos^2 x}{\sin^2 x}$$
$$= \frac{5x^4\,.\sin x - x^5\cos x + (\sin^2 x + \cos^2 x)}{\sin^2 x}$$
$$= \frac{5x^4\sin x - x^5\cos x + 1}{\sin^2 x}$$

Hence, the required answer is $\frac{5x^4\sin x - x^5\cos x + 1}{\sin^2 x}$.

191. Differentiate the following functions.

$$\sin^3 x\cos^3 x$$

Ans. : $\frac{d}{dx}(\sin^3 x\cos^3 x)$

$$= \sin^3 x \cdot \frac{d}{dx}\cos^3 x \cdot \frac{d}{dx}(\sin^3 x)$$
$$= \sin^3 x.3\cos^3 x(-\sin x) + \cos^3 x \cdot 3\sin^2 x \cdot \cos x$$
$$= -3\sin^4 x\cos^2 + 3\cos^4 x\sin^2 x$$
$$= 3\sin^2 x\cos^2 x(-\sin^2 x + \cos^2 x)$$
$$= 3\sin^2 x\cos^2 x \cdot \cos 2x$$
$$= \frac{3}{4} \cdot 4\sin^2 x\cos^2 x \cdot \cos 2x$$
$$= \frac{3}{4}(2\sin x\cos x)^2\cos 2x$$
$$= \frac{3}{4}\sin^2 2x \cdot \cos 2x$$

Hence, the required answer is $\frac{3}{4}\sin^2 2x \cdot \cos 2x$.

192. Evaluate:

$$\lim_{x \to \frac{\pi}{6}} \frac{\sqrt{3}\sin x - \cos x}{x - \frac{\pi}{6}}$$

Ans. : Given that $\lim\limits_{x \to \frac{\pi}{6}} \dfrac{\sqrt{3}\sin x - \cos x}{x - \frac{\pi}{6}}$

$$= \lim_{x \to \frac{\pi}{6}} \frac{2\left[\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right]}{x - \frac{\pi}{6}}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{2\left[\cos\frac{\pi}{6}\sin x - \sin\frac{\pi}{6}\cos x\right]}{x - \frac{\pi}{6}}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{2\sin\left(x - \frac{\pi}{6}\right)}{\left(x - \frac{\pi}{6}\right)} \left[\therefore \lim_{x \to \frac{\pi}{6}} \frac{\sin x}{x} = 1\right]$$

$= 2.1 = 2$

Hence, the required answer is 2.

193. Differentiate the following functions.

$$\frac{x^2\cos\frac{\pi}{4}}{\sin x}$$

Ans. : $\dfrac{d}{dx}\left(\dfrac{x^2\cos\frac{\pi}{4}}{\sin x}\right)$

$$= \cos\frac{\pi}{4}.\frac{d}{dx}\left(\frac{x^2}{\sin x}\right)$$

$$= \frac{\frac{1}{\sqrt{2}}\left[\sin x.\frac{d}{dx}(x)^2 - x^2.\frac{d}{dx}(\sin x)\right]}{\sin^2 x}$$

$$= \frac{1}{\sqrt{2}}\left[\frac{\sin x 2x - x^2\cos x}{\sin^2 x}\right]$$

$$= \frac{1}{\sqrt{2}}\left[\frac{2x}{\sin x} - \frac{x^2\cos x}{\sin^2 x}\right]$$

$$= \frac{1}{\sqrt{2}}\left[2x\ \mathrm{cosec}x - x^2\ \cot x\ \mathrm{cosec}x\right]$$

$$= \frac{x}{\sqrt{2}}\mathrm{cosec}x\left[2 - x\cot x\right]$$

Hence, the required answer is $\dfrac{x}{\sqrt{2}}\mathrm{cosec}x\left[2 - x\cot x\right]$.

*** Given section consists of questions of 5 marks each.** **[40]**

194. Evaluate the following limits.

$$\lim_{x \to 0} \frac{(\sin(\alpha+\beta)x + \sin(\alpha-\beta)x + \sin 2\alpha \cdot x}{\cos 2\beta x - \cos 2\alpha x}$$

Ans. : Given $\lim\limits_{x\to 0} \dfrac{(\sin\,(\alpha+\beta)x+\sin\,(\alpha-\beta)x+\sin 2\alpha\cdot x}{\cos 2\beta x-\cos 2\alpha x}$

$$= \lim_{x\to 0} \frac{[2\sin \alpha x\cdot\cos\beta x+\sin 2\alpha\cdot x]\cdot x}{2\sin\,(\alpha+\beta)x\cdot\sin\,(x-\beta)x}$$

$$= \lim_{x\to 0} \frac{[2\sin \alpha x\cdot\cos\beta x+2\sin\alpha x\cdot\cos\alpha x]\cdot x}{2\sin\,(\alpha+\beta)x\cdot\sin\,(x-\beta)x}$$

$$= \lim_{x\to 0} \frac{2\sin \alpha x(\cos\beta x+\cos\alpha x)\cdot x}{2\sin\,(\alpha+\beta)x\cdot\sin\,(\alpha-\beta)x}$$

$$= \lim_{x\to 0} \frac{\sin\alpha x\left[2\cos\,\left(\frac{\alpha+\beta}{2}\right)x\cdot\cos\,\left(\frac{\alpha-\beta}{2}\right)x\right]\cdot x}{\sin\,(\alpha+\beta)x\cdot\sin\,(\alpha-\beta)x}$$

$$= \lim_{x\to 0} \frac{\sin\alpha x\left[2\cos\,\left(\frac{\alpha+\beta}{2}\right)x\cdot\cos\,\left(\frac{\alpha-\beta}{2}\right)x\right]\cdot x}{2\sin\,\left(\frac{\alpha+\beta}{2}\right)x\cdot\cos\,\left(\frac{\alpha-\beta}{2}\right)x}$$

$$= \lim_{x\to 0} \frac{\sin\alpha x\cdot x}{2\sin\,\left(\frac{\alpha+\beta}{2}\right)x\cdot\cos\,\left(\frac{\alpha-\beta}{2}\right)x}$$

$$= \lim_{x\to 0} \frac{1}{2}\frac{\frac{\sin\alpha x}{\alpha x}\cdot(\alpha x)\cdot x}{\left[\frac{\sin\frac{\alpha+\beta}{2}x}{\frac{\alpha+\beta}{2}x}\times\left(\frac{\alpha+\beta}{2}\right)x\right]\left[\frac{\sin\,\left(\frac{\alpha-\beta}{2}\right)x}{\left(\frac{\alpha-\beta}{2}\right)x}\times\frac{(\alpha-\beta)}{2}x\right]}$$

$$= \frac{1}{2}\cdot\frac{\alpha x^2}{\left(\frac{\alpha+\beta}{2}\right)x\cdot\left(\frac{\alpha-\beta}{2}\right)x}$$

$$= \frac{1}{2}\left[\frac{\alpha}{\left(\frac{\alpha+\beta}{2}\right)\left(\frac{\alpha-\beta}{2}\right)}\right]$$

$$= \frac{1}{2}\cdot\frac{4\alpha}{\alpha^2-\beta^2}$$

$$= \frac{2\alpha}{\alpha^2-\beta^2}$$

Hence, the required amswer is $\dfrac{2\alpha}{\alpha^2-\beta^2}$.

195. Evaluate the following limits.

$$\lim_{x\to\pi}\frac{1-\sin\frac{x}{2}}{\cos\frac{x}{2}\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)}$$

Ans. : Given $\lim\limits_{x\to\pi}\dfrac{1-\sin\frac{x}{2}}{\cos\frac{x}{2}\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)}$

$$= \lim_{x\to\pi}\frac{\cos^2\frac{x}{4}+\sin^2\frac{x}{4}-2\sin\frac{x}{4}\cdot\cos\frac{x}{4}}{\cos\frac{x}{2}\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)}$$

$$= \lim_{x\to\pi}\frac{\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)^2}{\cos\frac{x}{2}\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)\left(\cos\frac{\pi}{4}-\sin\frac{x}{4}\right)}$$

$$= \lim_{x \to \pi} \frac{1}{\left(\cos \frac{\pi}{4} + \sin \frac{\pi}{4}\right)}$$

$$= \frac{1}{\cos \frac{\pi}{4} + \sin \frac{\pi}{4}}$$

$$= \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \frac{1}{\frac{2}{\sqrt{2}}}$$

$$= \frac{1}{\sqrt{2}}$$

Hence. the required answer is $\frac{1}{\sqrt{2}}$.

196. Evaluate:

$$\lim_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$$

Ans. : Given that $\lim_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$

$$= \lim_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} \times \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}}$$

$$= \lim_{x \to a} \frac{\left(2\cos\frac{x+a}{2} . \sin\frac{x-a}{2}\right)\sqrt{x} + \sqrt{a}}{x - a}$$

$$= \lim_{x \to a} \cos\left(\frac{x+a}{2}\right)\left(\sqrt{x} + \sqrt{a}\right)$$

Taking limits we have

$$= \cos\left(\frac{a+a}{2}\right)\left(\sqrt{a} + \sqrt{a}\right)$$

$$= \cos x \times 2\sqrt{a} = 2\sqrt{a}. \cos a$$

Hence, the required answer is $2\sqrt{a}. \cos a$.

197. Evaluate the following limits.

$$\lim_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$$

Ans. : Given $\lim_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\tan x(\tan^2 x - 1)}{\cos\left(x + \frac{\pi}{4}\right)}$$

$$= \lim_{x \to \frac{\pi}{4}} \tan x \cdot \lim_{x \to \frac{\pi}{4}} \left[\frac{(1 - \tan^2 x)}{\cos\left(x + \frac{\pi}{4}\right)}\right]$$

$$= -1 \times \lim_{x \to \frac{\pi}{4}} \frac{(1 - \tan x)(1 + \tan x)}{\cos\left(x + \frac{\pi}{4}\right)}$$

$$= -(1+1)\times \lim_{x\to\frac{\pi}{4}} \frac{(\cos x-\sin x)}{\cos x\cdot\cos\left(x+\frac{\pi}{4}\right)}$$

$$= -2\times \lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)}{\cos x\cdot\cos\left(x+\frac{\pi}{4}\right)}$$

$$= -2\sqrt{2}\lim_{x\to\frac{\pi}{4}} \frac{\left[\cos\frac{\pi}{4}\cdot\cos x-\sin\frac{\pi}{4}\sin x\right]}{\cos x\cdot\cos\left(x+\frac{\pi}{4}\right)}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{-2\sqrt{2}\cdot\cos\left(x+\frac{\pi}{4}\right)}{\cos x\cdot\cos\left(x+\frac{\pi}{4}\right)}$$

$$= \frac{-2\sqrt{2}}{\cos\frac{\pi}{4}}$$

$$= \frac{-2\sqrt{2}}{\frac{1}{\sqrt{2}}} = -2\times 2 = -4$$

Hence, the rquired answer is -4.

198. Evaluate the following limits.

$$\lim_{y\to 0} \frac{(x+y)\sec(x+y)-x\sec x}{y}$$

Ans. : $\lim_{y\to 0} \frac{(x+y)\sec(x+y)-x\sec x}{y}$

$$= \lim_{y\to 0} \frac{x\sec(x+y)+y\sec(x+y)-x\sec x}{y}$$

$$= \lim_{y\to 0} \frac{\left[x\sec(x+y)-x\sec x\right]}{y} + \lim_{y\to 0} \frac{y\sec(x+y)}{y}$$

$$= \lim_{y\to 0} \frac{x\left[\sec(x+y)-\sec x\right]}{y} + \lim_{y\to 0} \sec(x+y)$$

$$= \lim_{y\to 0} \frac{x\left[\frac{1}{\cos(x+y)}-\frac{1}{\cos x}\right]}{y} + \lim_{y\to 0} \sec(x+y)$$

$$= \lim_{y\to 0} x\left[\frac{\cos x-\cos(x+y)}{y\cdot\cos(x+y)\cdot\cos x}\right] + \lim_{y\to 0} \sec(x+y)$$

$$= \lim_{y\to 0} \frac{x\left[-2\sin\left(\frac{x+x+y}{2}\right)\cdot\sin\left(\frac{x-x-y}{2}\right)\right]}{y\cos(x+y)\cdot x} + \lim_{y\to 0} \sec(x+y)$$

$$= \lim_{y\to 0} \frac{x\left[-2\sin\left(x+\frac{y}{2}\right)\cdot\left(\frac{-y}{2}\right)\right]}{\cos(x+y)\cdot\cos x\cdot y} + \lim_{y\to 0} \sec(x+y)$$

Taking the limits we have

$$= x\left[\sin x\cdot\frac{1}{\cos x\cdot\cos x}\right] + \sec x$$

$$= x\sec x\tan x + \sec x$$

$= \sec x(x\tan x + 1)$

Hence, the required answer is $\sec x(x\tan x + 1)$.

199. Evaluate the following limits.

Let $\dfrac{k\cos x}{\pi - 2x},$ $\{3, x = \dfrac{\pi}{2}$ and $f(x) = f(\dfrac{\pi}{2})$ Find the value of k.

Ans. : Given $\begin{cases} \dfrac{k\cos x}{\pi - 2x}, \\ 3, x = \dfrac{\pi}{2} \end{cases}$

$$\text{L. H. L. } f(x) = \lim_{x \to \frac{\pi}{2}} \frac{k\cos x}{\pi - 2x} = \lim_{h \to 0} \frac{k\cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)}$$

$$= \lim_{h \to 0} \frac{k\sin h}{\pi - \pi + 2h} = \lim_{h \to 0} \frac{k\sin h}{2h}$$

$$= \frac{k}{2}.1 = \frac{k}{2}$$

$$\text{R. H. L. } f(x) = \lim_{x \to \frac{\pi}{2}} \frac{k\cos x}{\pi - 2x} = \lim_{h \to 0} \frac{k\cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)}$$

$$= \lim_{h \to 0} \frac{k\sin h}{\pi - \pi - 2h} = \lim_{h \to 0} \frac{-k\sin h}{-2h}$$

$$= \frac{k}{2}.1 = \frac{k}{2}$$

we are given that $\lim_{h \to \frac{\pi}{2}} f(x) = 3$

Hence, the required answer is 6.

200. Evaluate:

$$\lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$$

Ans. : Given that $\lim_{x \to 0} \dfrac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$

$$= \lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x} \times \frac{\sqrt{2} + \sqrt{1 + \cos x}}{\sqrt{2} + \sqrt{1 + \cos x}}$$

$$= \lim_{x \to 0} \frac{2 - (1 + \cos x)}{\sin^2 x\left[\sqrt{2} + \sqrt{1 + \cos x}\right]}$$

$$= \lim_{x \to 0} \frac{1 + \cos x}{\sin^2 x\left[\sqrt{2} + \sqrt{1 + \cos x}\right]}$$

$$= \lim_{x \to 0} \frac{2\sin^2 \frac{x}{2}}{(2\sin \frac{x}{2}\cos \frac{x}{2})^2} \times \frac{1}{\sqrt{2} + \sqrt{1 + \cos x}}$$

$$= \lim_{x \to 0} \frac{2\sin^2 \frac{x}{2}}{4\sin^2 \frac{x}{2}\cos^2 \frac{x}{2}} \times \frac{1}{\sqrt{2} + \sqrt{1 + \cos x}}$$

$$= \lim_{x \to 0} \frac{2}{4\cos^2 \frac{x}{2}} \times \frac{1}{\sqrt{2} + \sqrt{1 + \cos x}}$$

Taking limit, we get

$$= \frac{2}{4\cos^2 0} \times \frac{1}{(\sqrt{2} + \sqrt{2})}$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

Hence, the required answer is $\frac{1}{4\sqrt{2}}$.

201. Evaluate:

$$\lim_{x \to 0} \frac{2\sin x - \sin 2x}{x^3}$$

Ans. : Given that $\lim_{x \to 0} \frac{2\sin x - \sin 2x}{x^3}$

$$= \lim_{x \to 0} \frac{2\sin x - 2\sin x\cos x}{x^3}$$

$$= \lim_{x \to 0} \frac{2\sin x - (1 - \cos x)}{x^3}$$

$$= \lim_{x \to 0} \frac{2\sin x}{x}\left(\frac{1 - \cos x}{x^2}\right)$$

$$= \lim_{x \to 0} 2\left(\frac{\sin x}{x}\right)\left(\frac{\frac{2\sin^2 x}{2}}{x^2}\right)$$

$$= \lim_{x \to 0} 2\left(\frac{\sin x}{x}\right)\left(2\frac{\sin^2 \frac{x}{2}}{\frac{x^4}{4}} \times \frac{1}{4}\right)$$

$$= \lim_{x \to 0} 2\left(\frac{\sin x}{x}\right)\left(2\frac{\sin^2 \frac{x}{2}}{\frac{x^4}{4}}\right)^2 . \frac{1}{4}$$

$$= \lim_{x \to 0} \frac{4}{4}\left(\frac{\sin x}{x}\right) \lim_{\frac{x}{2} \to 0}\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)$$

$1.1.(1)^2 = 1$

Hence, the required answer is 1.

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